ARIZONA STATE RETIREMENT SYSTEM

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Paul Matson
Director

AGENDA

NOTICE OF POSSIBLE QUORUM AND PUBLIC MEETING OF THE ARIZONA STATE RETIREMENT SYSTEM BOARD OR OPERATIONS AND AUDIT COMMITTEE OR EXTERNAL AFFAIRS COMMITTEE OR INVESTMENT COMMITTEE

3300 North Central Avenue
14th Floor Conference Room
Phoenix, Arizona 85012

March 26, 2015
9:00 a.m.

Pursuant to A.R.S. § 38-431.02, notice is hereby given to the Trustees of the Arizona State Retirement System (ASRS) Board and to the general public that the ASRS will hold a Educational Session on Thursday, March 26, 2015, beginning at 9:00 a.m., in the 14th Floor Conference Room of the ASRS offices at 3300 N. Central Avenue, Phoenix, Arizona 85012

A possible quorum of the ASRS Board, or Operations and Audit Committee, or External Affairs Committee, or Investment Committee could occur, so the meeting will be open to the public. Trustees of the Board may attend either in person or by telephone conference call.

This meeting is an Educational Seminar. The ASRS Board, Operations and Audit Committee, External Affairs Committee, or Investment Committee will NOT be in attendance to discuss business of the public body, nor will any action of the ASRS Board, Operations and Audit Committee, External Affairs Committee, or Investment Committee be taken at this meeting.

This meeting will NOT be teleconferenced to the ASRS Tucson office at 7660 East Broadway Boulevard, Suite 108, Tucson, Arizona 85710.

The Agenda for the meeting is as follows:

1. Presentation and Discussion Regarding Understanding the Low Volatility Anomaly *(Informational item)* .....Dr. Brendan Bradley
   Senior Vice President, Director and Portfolio Manager, Acadian
   .....Dr. Seth Weingram
   Senior Vice President, Acadian
   .....Mr. Andrew D. Miller
   Vice President, Acadian

2. Presentation and Discussion Regarding The Theory Behind Using Systematic Rebalancing to Capture Natural Stock price Volatility *(Informational item)* ..... Dr. Adrian Banner
Chief Investment Officer, Intech
.....Mr. Christian McCormick
Managing Director, Intech
.....Mr. Warren DeKinder
Managing Director, Intech

3. Adjournment.

A copy of the agenda background material provided to Committee Trustees (with the exception of material relating to possible executive sessions) is available for public inspection at the ASRS offices located at 3300 North Central Avenue, 14th Floor, Phoenix, Arizona, and 7660 East Broadway Boulevard, Suite 108, Tucson, Arizona. The agenda is subject to revision up to 24 hours prior to meeting. These materials are also available on the ASRS website (https://www.azasrs.gov/web/BoardCommittees.do) approximately 48 hours prior to the meeting.

Persons with a disability may request a reasonable accommodation such as a sign language interpreter or alternate formats of this document by contacting Tracy Darmer, ADA Coordinator at (602) 240-5378 in Phoenix, at (520) 239-3100, ext. 5378 in Tucson or 1-800-621-3778, ext. 5378 outside metro Phoenix or Tucson. Requests should be made as early as possible to allow time to arrange the accommodations.

Dated March 23, 2015

ARIZONA STATE RETIREMENT SYSTEM

Gloria Trujillo Committee Administrator	Gary Dokes Chief Investment Officer

Agenda Item #1

Acadian Biographies

Brendan O. Bradley, Ph.D. – Senior Vice President, Director, Portfolio Management

Brendan joined Acadian in September 2004 as a senior member of the Research and Portfolio Management Team. In 2010, Brendan was appointed Director of our Managed Volatility Strategies, and in 2013 became Director, Portfolio Management overseeing portfolio management policy. He is a member of the Acadian Executive Committee and Operating Committee. Prior to Acadian, Brendan was a vice president at Upstream Technologies, where he designed and implemented investment management systems and strategies. His professional background also includes work as a research analyst and consultant at Samuelson Portfolio Strategies. Education: B.A., Physics, Boston College; Ph.D., Applied Mathematics, Boston University.

Brendan O. Bradley, Ph.D. – Senior Vice President, Director, Portfolio Management
§ 16 years of investment experience
§ Member of the Acadian Executive Committee and Operating Committee
§ Ph.D. in Applied Mathematics from Boston University
§ B.A. in Physics from Boston College

Seth Weingram, Ph.D. – Senior Vice President, Strategist

Seth joined Acadian in December 2014 in the role of strategist, aligned closely with Acadian's Marketing & Client Service and Investment teams. Seth brings 17 years of financial industry experience to the firm. Most recently, he was a managing director in Equity Derivatives Trading at UBS, where he ran the electronic options market making desk, led development of quantitative trading tools, and launched Equity Derivatives Strategy. Previously, Seth was a researcher at Barclays Global Investors focusing on options and volatility, and he was a founding member and for several years global head of Deutsche Bank’s award-winning Equity Derivatives Strategy Group. Seth holds a Ph.D. in Economics from Stanford University and a B.A. in Economics from the University of Chicago.

Seth Weingram – Senior Vice President, Strategist
§ 17 years of professional experience
§ Ph.D. in Economics from Stanford University
§ B.A. in Economics from University of Chicago

Andrew D. Miller – Vice President

Andrew joined Acadian in February 2004 and is a member of Acadian's Marketing and Client Service Team. He has worked in Client Service, Marketing and Operations in his career at Acadian. Previously, he worked in global custody at State Street Corporation. Education: B.A., Wheaton College; M.B.A., Northeastern University.

Andrew D. Miller – Vice President
§ 14 years of professional experience
§ M.B.A. from Northeastern University
§ B.A. from Wheaton College

ACADIAN

MANAGED VOLATILITY STRATEGIES

HISTORICAL EVIDENCE

BEHAVIORAL EXPLANATION

CAPTURING THE MISPRICING

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AGENDA

INTRODUCTION What Is “Managed Volatility”?

HISTORICAL EVIDENCE Risk And Return Around The World

BEHAVIORAL EXPLANATION Will The Low-risk Anomaly Persist?

EXECUTION Capturing The Mispricing

APPLICATION Asset Allocation

CONCLUSION

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INTRODUCTION

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WHAT IS MANAGED VOLATILITY?

- Finance theory say that investors who buy higher risk stocks will be compensated with higher returns
- The empirical reality is that the relationship between measures of risk and returns is hard to find
- Simple conclusion: Combine lower risk stocks into a managed volatility portfolio

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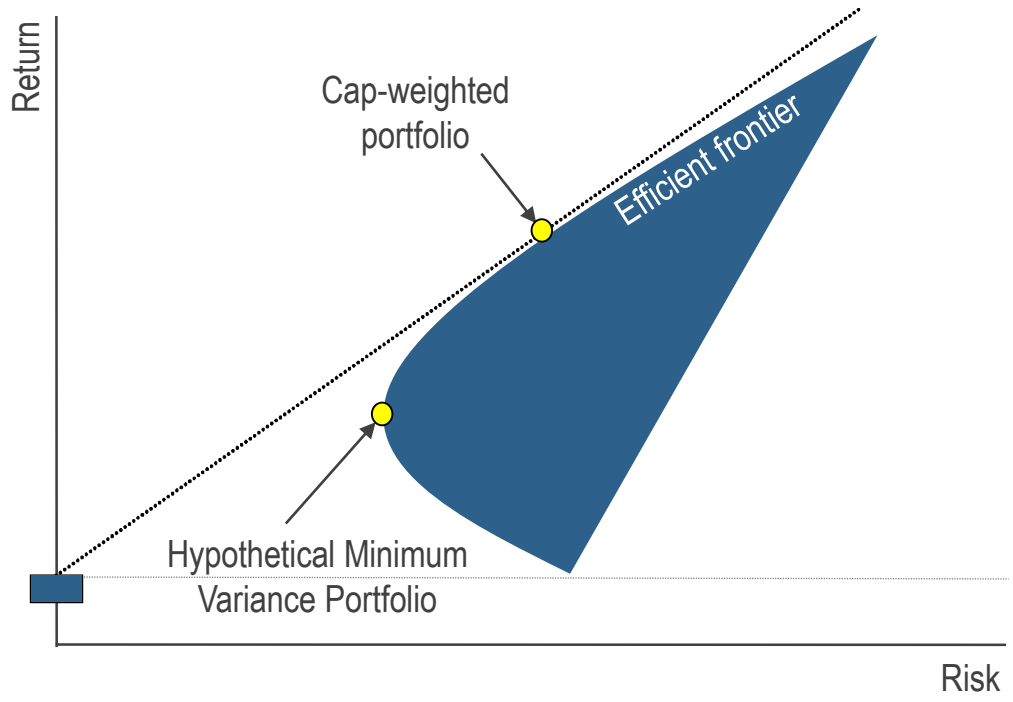
WHAT ARE MANAGED VOLATILITY STRATEGIES?



Finance theory says the market portfolio should lie on the efficient frontier and outperform the minimum variance portfolio.

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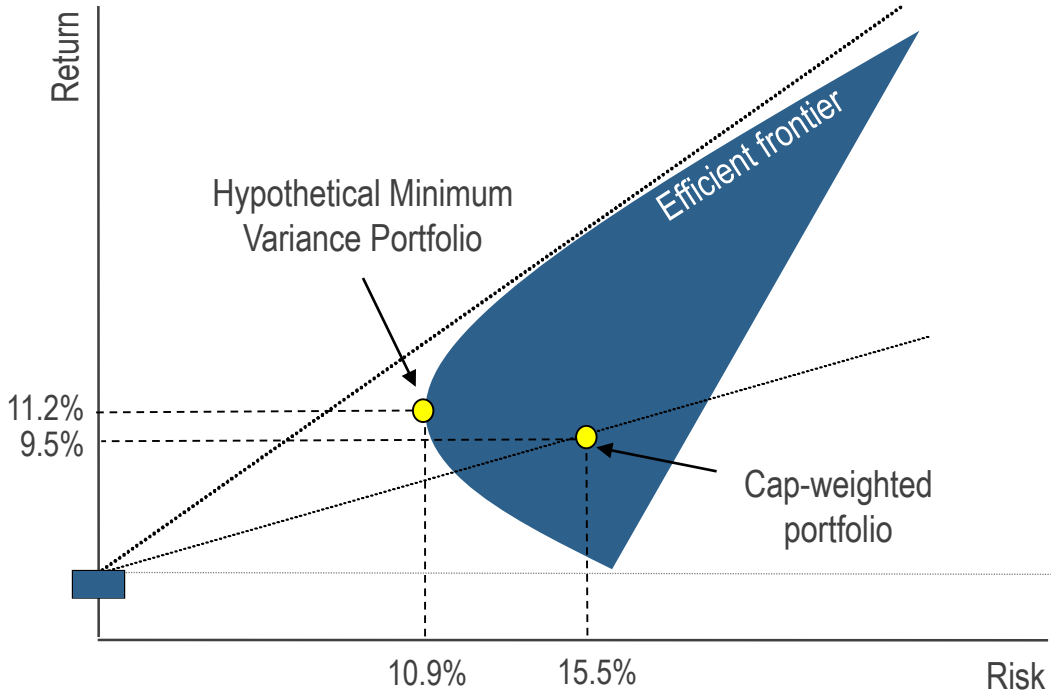
ANNUALIZED VOLATILITY, RETURN FOR U.S. STOCKS

1968 – 2012



The empirical reality is that minimum variance portfolios have outperformed the market, but with less risk

Source: Acadian Asset Management LLC, CRSP®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. crsp.uchicago.edu
For illustrative purposes only. This is meant to be an educational illustrative example and is not intended to represent investment returns generated by an actual portfolio. They do not represent actual trading or an actual account, but were achieved by means of using the CRSP universe of securities as a whole. Results do not reflect transaction costs, other implementation costs and do not reflect advisory fees or their potential impact. Hypothetical results are not indicative of actual future results. Every investment program has the opportunity for loss as well as profit.

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HISTORICAL EVIDENCE

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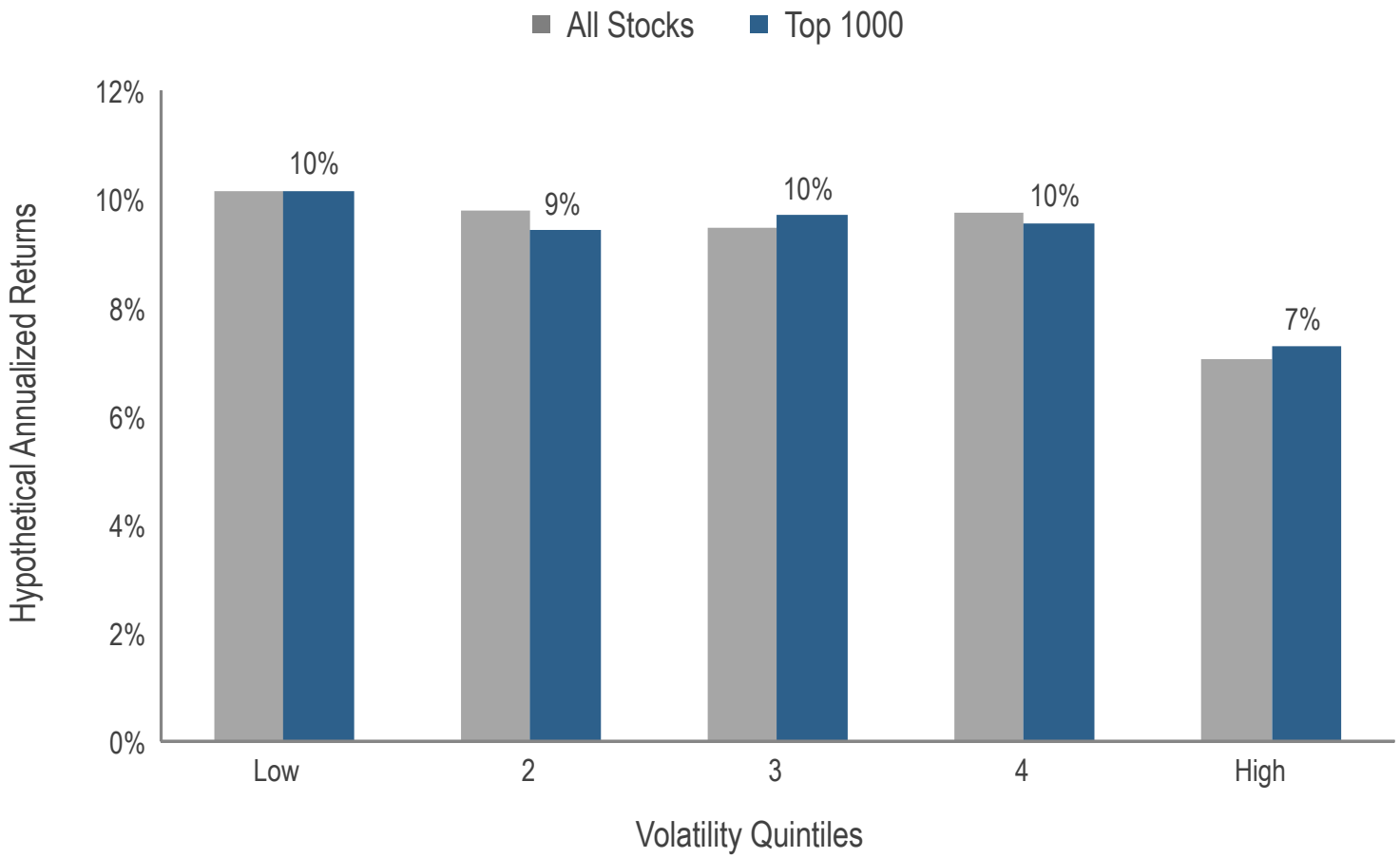
U.S. EVIDENCE: LOWER RISK STOCKS HAVE EARNED HIGHER AVERAGE RETURNS

1968 – 2012



Source: Acadian Asset Management LLC, CRSP®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. crsp.uchicago.edu
Methodology: "All Stocks" refers to all of the U.S. stocks within the CRSP Universe. "Top 1000" refers to the top 1000 capitalized U.S. stocks within the CRSP Universe. Equal sized quintiles, cap weighted, from 1968 through 2012.
For illustrative purposes only. This is meant to be an educational illustrative example and is not intended to represent investment returns generated by an actual portfolio. They do not represent actual trading or an actual account, but were achieved by means of using the CRSP universe of securities as a whole. Results do not reflect transaction costs, other implementation costs and do not reflect advisory fees or their potential impact. Hypothetical results are not indicative of actual future results. Every investment program has the opportunity for loss as well as profit.

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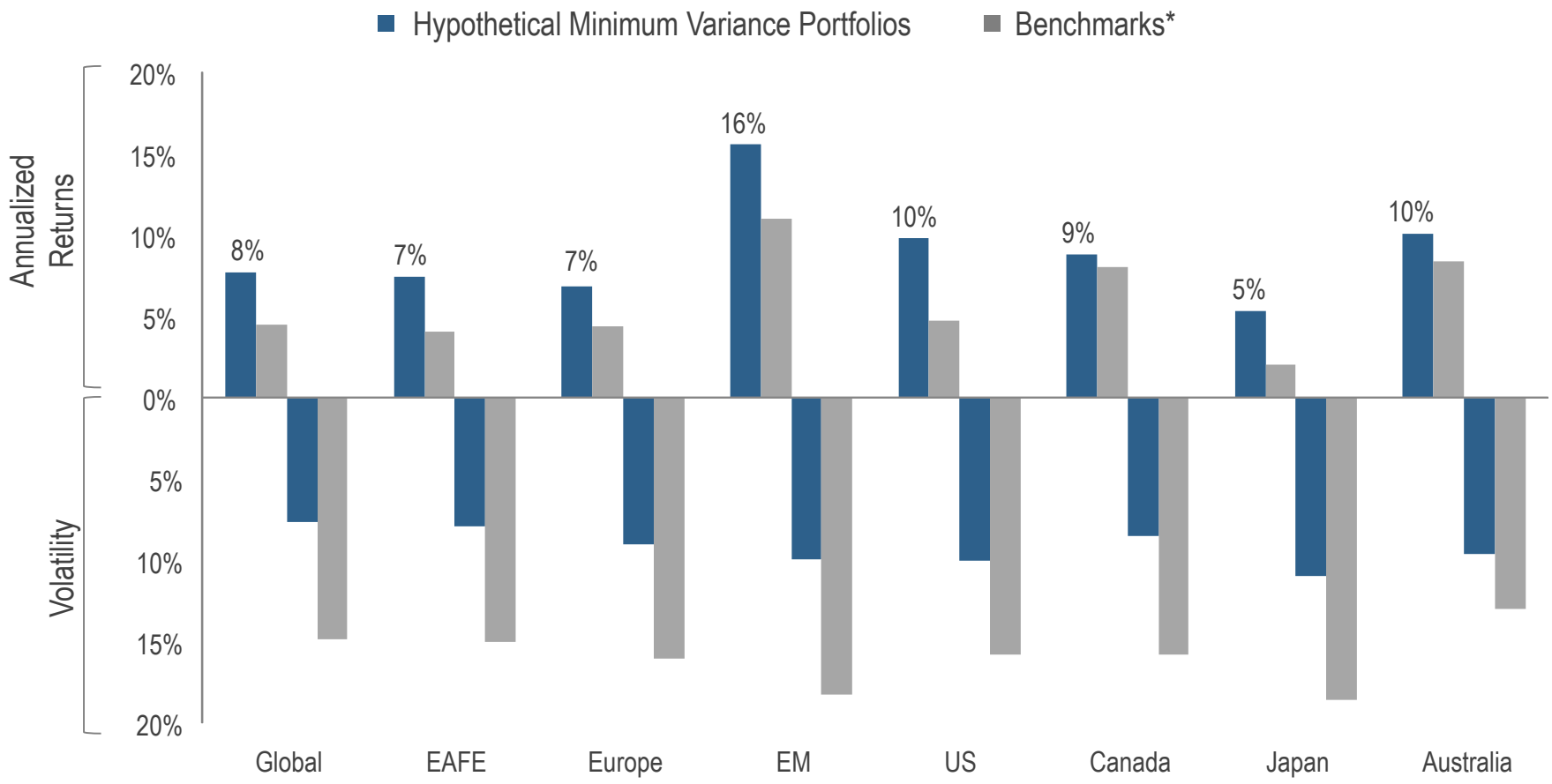
THE LOW VOLATILITY ANOMALY IS APPARENT AROUND THE WORLD

1998 – 2013



Source: Acadian Asset Management LLC, *MSCI World, MSCI Europe, MSCI EAFE, MSCI US and MSCI JP.
Methodology: Hypothetical portfolios period covers October 1998-September 2013, except EM, which covers January 2004 – September 2013.
For illustrative purposes only. This is meant to be an educational illustrative example and is not intended to represent investment returns generated by an actual portfolio. The Hypothetical Minimum Variance Portfolios we have created for educational illustrations do not include constraints on exposures to industries or, where relevant, countries and regions, nor do they manage toward a target for total portfolio risk. The hypothetical results do not represent actual trading or an actual account, but were achieved by means of retroactive application of a model designed with the benefit of hindsight for the period specified above. Results are gross and would be reduced by advisory fees. Results do not reflect transaction costs, other implementation costs and do not reflect advisory fees or their potential impact. Reference to the benchmark is for comparative purposes only. Hypothetical performance is not indicative of actual future results. Every investment program has the opportunity for loss as well as profit. Index Source: MSCI Copyright MSCI 2015. All Rights Reserved. Unpublished. PROPRIETARY TO MSCI.

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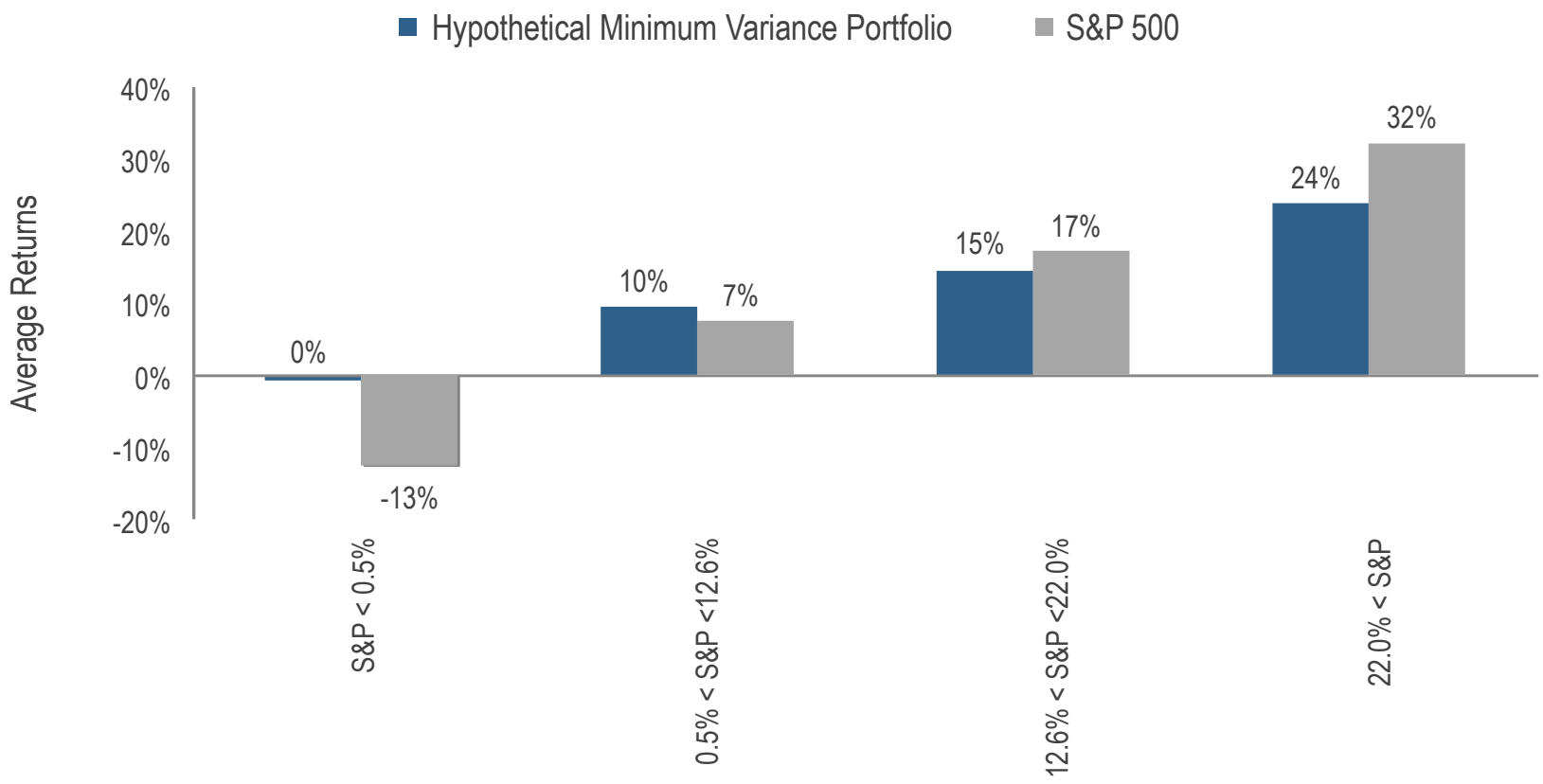
PERFORMANCE UNDER VARIOUS MARKET CONDITIONS

1969 – 2012



Source: Acadian Asset Management LLC, CRSP,CRSP®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. crsp.uchicago.edu, S&P Universe of Securities.

Methodology: S&P Return Quartiles, Rolling 12 Month Returns, 1969 – 2012. For illustrative purposes only. This is meant to be an educational illustrative example and is not intended to represent investment returns generated by an actual portfolio. They do not represent actual trading or an actual account, but were achieved by means of using the CRSP universe of securities as a whole. Results do not reflect transaction costs, other implementation costs and do not reflect advisory fees or their potential impact. Hypothetical results are not indicative of actual future results. Every investment program has the opportunity for loss as well as profit.

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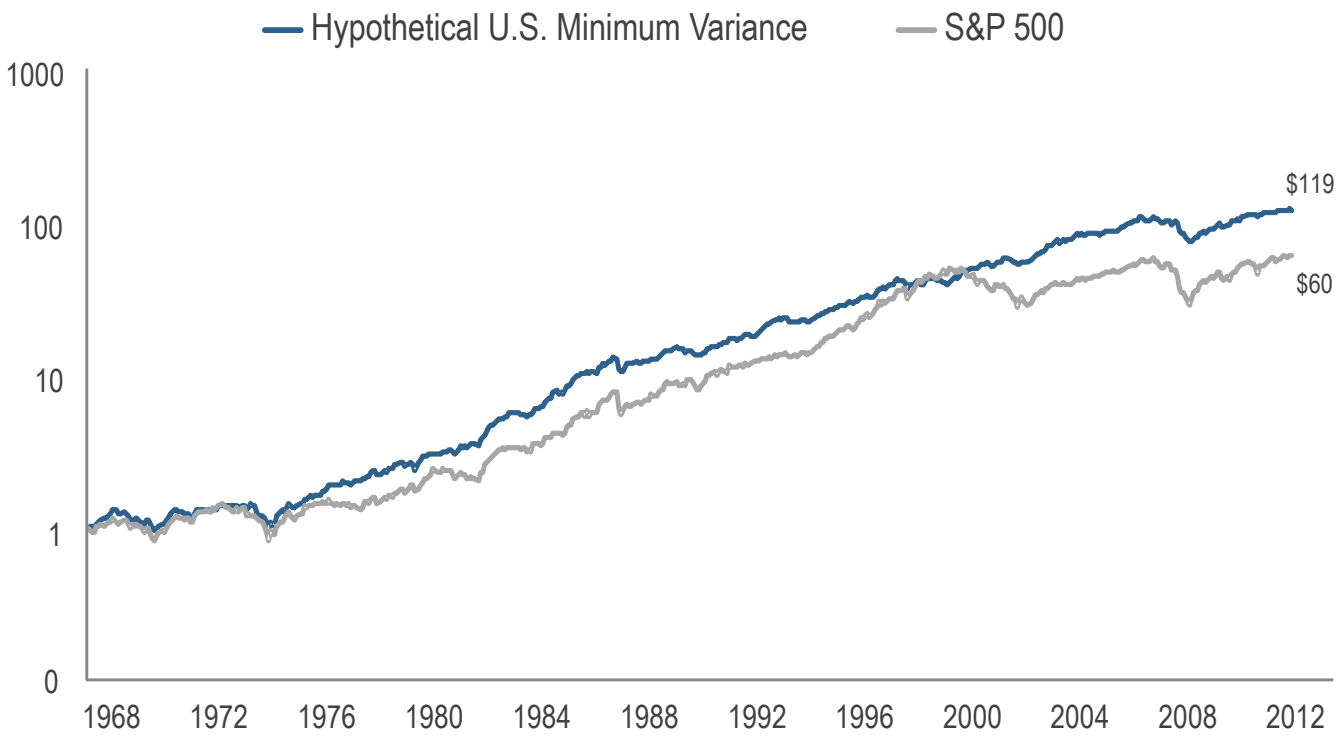
LONG-TERM RETURNS: U.S. MINIMUM VARIANCE

1968 – 2012



A minimum variance portfolio compounds to $119, versus $60 for the S&P 500.

Returns are cumulative. Source: Acadian Asset Management LLC, CRSP®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. crsp.uchicago.edu. Index Source: Copyright © 2015, Standard & Poor's Financial Services LLC. All rights reserved. Methodology: Hypothetical portfolio period covers January 1968 – December 2012.
For illustrative purposes only. This is meant to be an educational illustrative example and is not intended to represent investment returns generated by an actual portfolio. They do not represent actual trading or an actual account, but were achieved by means of using the CRSP universe of securities as a whole. Results do not reflect transaction costs, other implementation costs and do not reflect advisory fees or their potential impact. Hypothetical results are not indicative of actual future results. Every investment program has the opportunity for loss as well as profit.

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OBSERVATIONS

- Low-risk stocks have realized high average returns
- Low-variance returns compound to higher cumulative returns
- High Sharpe Ratio for managed volatility portfolios (not reflected in Information Ratio)

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BEHAVIORAL EXPLANATION

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WHY A LOW-VOLATILITY ANOMALY?

Behavioral explanations have two ingredients

1. Investor psychology
2. Limits to arbitrage

1. Psychology: Investors are attracted to risky stocks

- Overconfidence
- Representativeness
- Lottery preferences

2. Limits to arbitrage: Institutional investors are constrained

- Handcuffed to index benchmarks
- Buying a low-beta stock creates tracking-error risk

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OVERCONFIDENCE

- In one psychological study, young adults were asked to rank themselves.

- When asked to rate “the ability to get along with others”
  - **100%** put themselves in the top half of the population
  - **60%** rated themselves in the top 10% of the population
  - **25%** humbly thought they were in the top 1% of the population

Myers, D.G. 1980. The inflated self. NY. Seabury Press

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OVERCONFIDENCE

- In one psychological study, young adults were asked to rank themselves.
- When asked about leadership
  - **70%** rated themselves in the top quartile in leadership
  - Only **2%** felt they were below average

Myers, D.G. 1980. The inflated self. NY. Seabury Press

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OVERCONFIDENCE

- In one psychological study, young adults were asked to rank themselves.
- Regarding athletic ability
  - **60%** said they were in the top quartile of athletic ability
  - Only **6%** said they were below average

Myers, D.G. 1980. The inflated self. NY. Seabury Press

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OVERCONFIDENCE

General knowledge questions	Answers
1. Three fourths of the world's cacao comes from where?	Africa or South America
2. Which causes more deaths in the U.S.?	Appendicitis or pregnancy & child birth
3. When was the first air raid?	1849 or 1937
4. Adonis was the god of?	Love or vegetation
5. Khalil Gibran was most inspired by which religion?	Buddhist or Christian
6. Dido and Aeneas is an opera written by whom?	Berlioz or Purcell
7. Potatoes are native to where?	Ireland or Peru

Fischhoff, B., Slovic, P., & Lichtenstein, S. (1977). "Knowing with certainty: the appropriateness of extreme confidence"., Journal of Experimental Psychology: Human Perception and Performance Volume 3, Issue 4, November 1977, Pages 552-564

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OVERCONFIDENCE

Answers	Correct	Cases of extreme overconfidence*
1. Africa	4.8%	35.7%
2. Appendicitis	19.0%	30.9%
3. 1849	26.2%	23.8%
4. Vegetation	31.0%	23.8%
5. Christian	33.3%	16.6%
6. Purcell	33.3%	4.8%
7. Peru	35.7%	23.8%

*The number of subjects was 42. Fischhoff, B., Slovic, P., & Lichtenstein, S. (1977). "Knowing with certainty: the appropriateness of extreme confidence"., Journal of Experimental Psychology: Human Perception and Performance Volume 3, Issue 4, November 1977, Pages 552-564

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REPRESENTATIVENESS

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

Which of the two alternatives is more probable?

A) Linda is a bank teller

B) Linda is a bank teller and is active in the feminist union

Kanhman, D. and Tversky, A. (1983), "Extensional verse intuitive reasoning: The conjunction fallacy in probable judgment:, Psychological Review, 90, 293-315

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REPRESENTATIVENESS

List 3 great stock investments over the last 20 years

List 3 horrible stocks over the last 20 years

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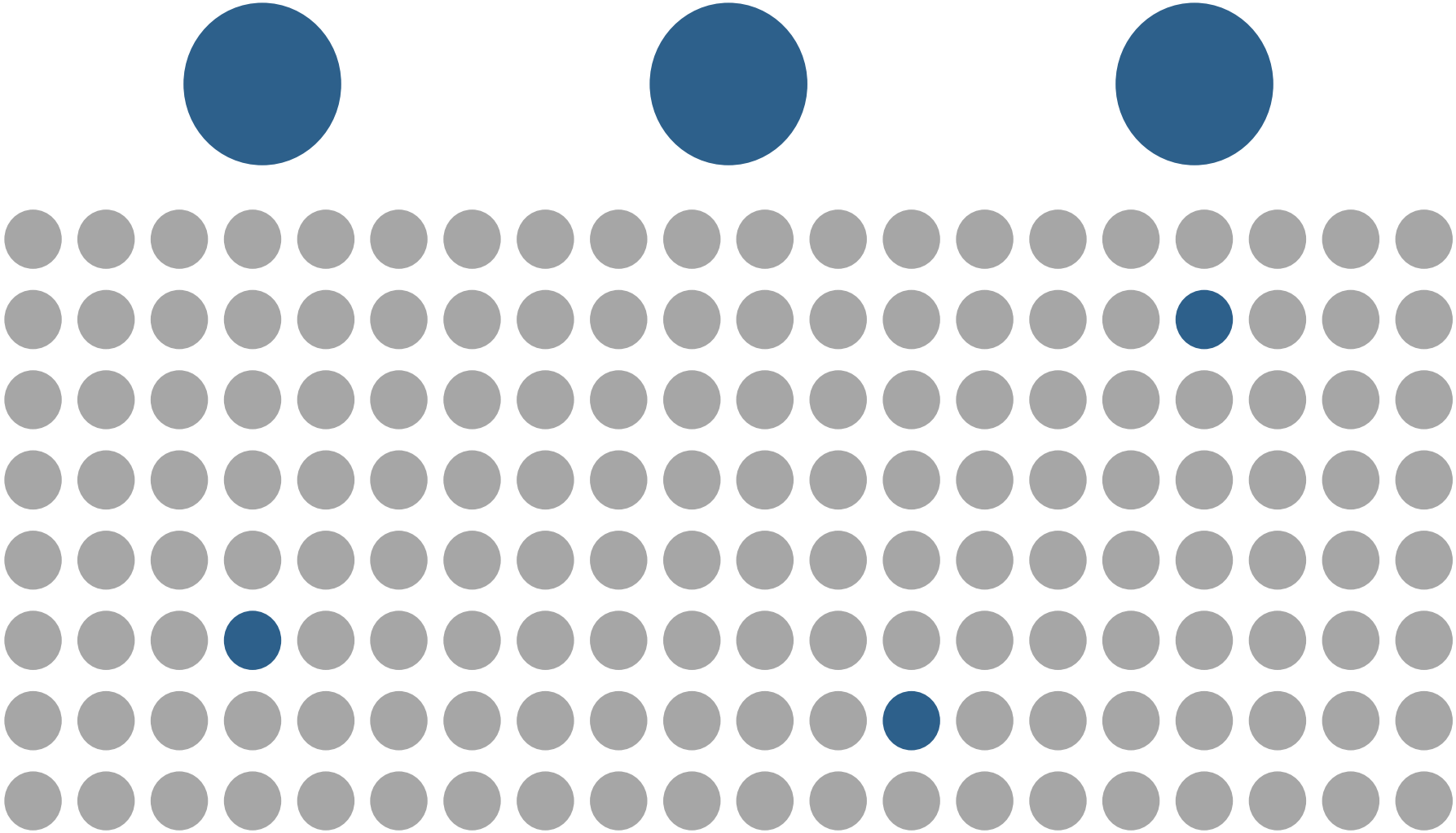
REPRESENTATIVENESS



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STOCK AS LOTTERIES

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STOCK AS LOTTERIES

Test #1

Choose one of the following:

1. (6,000, p=0.45)
2. (3,000, p=0.90)

Kanhman, D. and Tversky, A. (1979), "Prospect Theory: An Analysis of Decision under Risk," Econmetrica, 47, 263-292.

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STOCK AS LOTTERIES

Test #2

Choose one of the following:

1. (6,000, p=0.001)
2. (3,000, p=0.002)

Kanhman, D. and Tversky, A. (1979), "Prospect Theory: An Analysis of Decision under Risk," Econmetrica, 47, 263-292.

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STOCK AS LOTTERIES

Test #1

Choose one of the following:

a) (6,000, p=0.45)

N=66 14%

b) (3,000, p=0.90)

86%

Test #2

Choose one of the following:

c) (6,000, p=0.001)

N=66 73%

d) (3,000, p=0.002)

27%

Kanhman, D. and Tversky, A. (1979), "Prospect Theory: An Analysis of Decision under Risk," Econmetrica, 47, 263-292.

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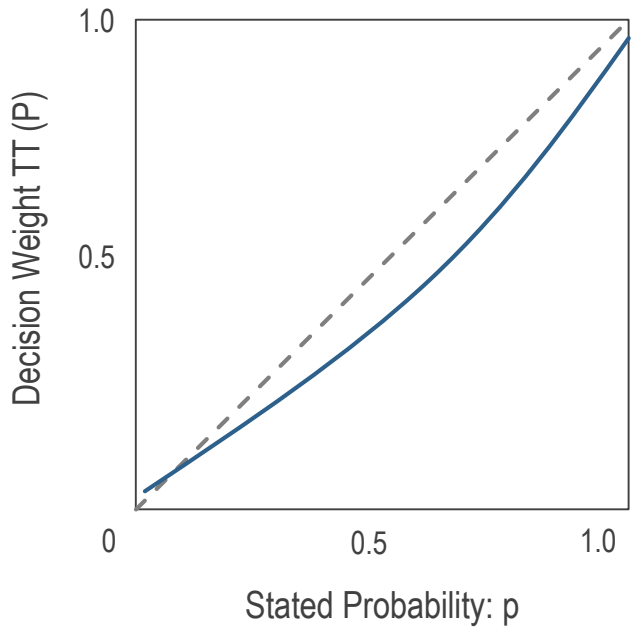
STOCK AS LOTTERIES

- For unlikely events, people overweight the probability
- Thus, people are willing to overpay for a small chance to get rich
- Lottery tickets have negative expected value



Kanhman, D. and Tversky, A. (1979), "Prospect Theory: An Analysis of Decision under Risk," Econmetrica, 47, 263-292.

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STOCK AS LOTTERIES

Do people treat stocks like lottery tickets? In Taiwan they seem to:

Taiwan lottery is extremely popular

- 8% of government budget comes from lottery
- Absenteeism on lottery days (“paralyzed”)

Taiwan stock and lotteries are substitutes

- Big lottery jackpots => substantially lower trading volume
- Effect is strongest in stocks with lottery features such as high return volatility and skewness

Treating stocks as lotteries creates mispricing

- Individual investors lose 2.2% of GDP/year
- Individuals lose, institutions win

Barber, B. M., Lee, Y.T.. And Odean, T. “Just How Much Do Individuals Lose by Trading?” 2006.
Goa, Xiaohui and Lin, Tse-Chun, Do Behavior Needs Influence the Trading Activity of Individual Investors Evidence Repeated Natural Experiments (July 15, 2010). Available at SSRN: http://ssrn.com/abstract=1622184

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STOCK AS LOTTERIES

- More generally, if people view some stocks as lottery tickets, then those stocks will have negative expected value
- People who own a small number of stocks – 80% of individuals portfolios have fewer than 5 stocks
- Investors buy stocks that look like lottery tickets (positively skewed)

Barberis, N. and M. Huang (2007). "Stocks as Lotteries: The Implications of Probably Weighting for Security Prices ," NBER working paper.
Polkovnichenko, V., 2005, "Household Portfolio Diversification: A Case for Rank- dependent Preferences," Review of Financial Studies, 18, 1467-1502.
Mitton, T. and K. Vorkink (2007). "Equilibrium Underdiversification and the Preference for Skewness." Review of Financial Studies 20(4):1255-12-88

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WILL THE LOW RISK ANOMALY PERSIST?

BEHAVIORAL FINANCE FOUNDATIONS

Overconfidence

- Investors are overconfident in their forecasts
- More volatile stocks generate more disagreement, and more extreme optimists… optimists drive prices with short sales constraints

Representativeness*

- Investors emphasize salient stories, like Microsoft, Whole Foods, Genzyme as representative of risky stocks, underweighting the high priori likelihood of failure for similar firms

Stocks As Lotteries

- Volatile stocks, given limited liability, are positively skewed
- Investors accept a high probability of loss, low probability of jackpot

*Not a recommendation to buy or sell a specific security

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WHY DOESN'T SMART MONEY CAPITALIZE?

- Mandates often focus on Information Ratio

- An explicit or implicit Information Ratio mandate handcuffs the institutional investment manager…

  See Baker, Bradley, and Wurgler (2011) for details*

In a nutshell: to a cap-weighted benchmark-sensitive investor, low-volatility stocks are risky

*"Benchmarks as Limits to Arbitrage." Financial Analysts Journal 67.1 (2011): 40-54

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WILL THE LOW RISK ANOMALY PERSIST?

Yes, as long as:

- Behavioral foundations remain intact
- Institutional managers remain benchmark constrained

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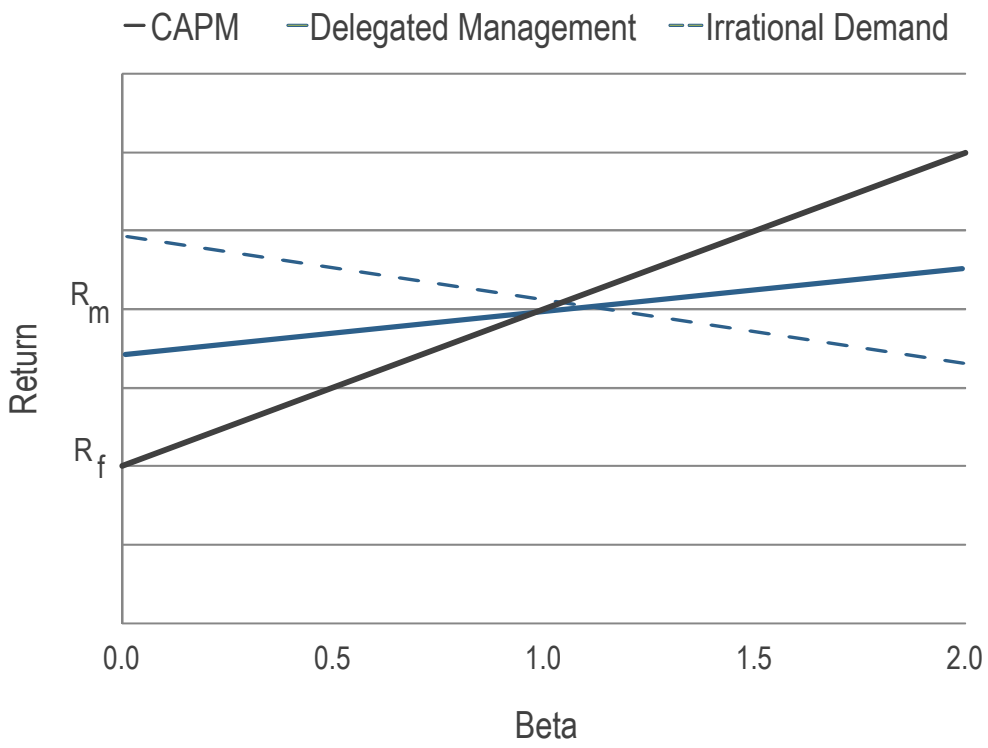
BENCHMARKS ELIMINATE THE LINK BETWEEN RISK AND RETURN

- Consider overweighting a high alpha, low beta stock
- An easy decision with a maximum Sharpe Ratio mandate
- But it may not be a good bet from the perspective of maximizing Information Ratio



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EXECUTION

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THE MECHANICS OF MANAGED VOLATILITY PORTFOLIOS

- Carefully manage risk
- Consider transaction costs and other dimensions in portfolio construction
- Incorporate return forecasts

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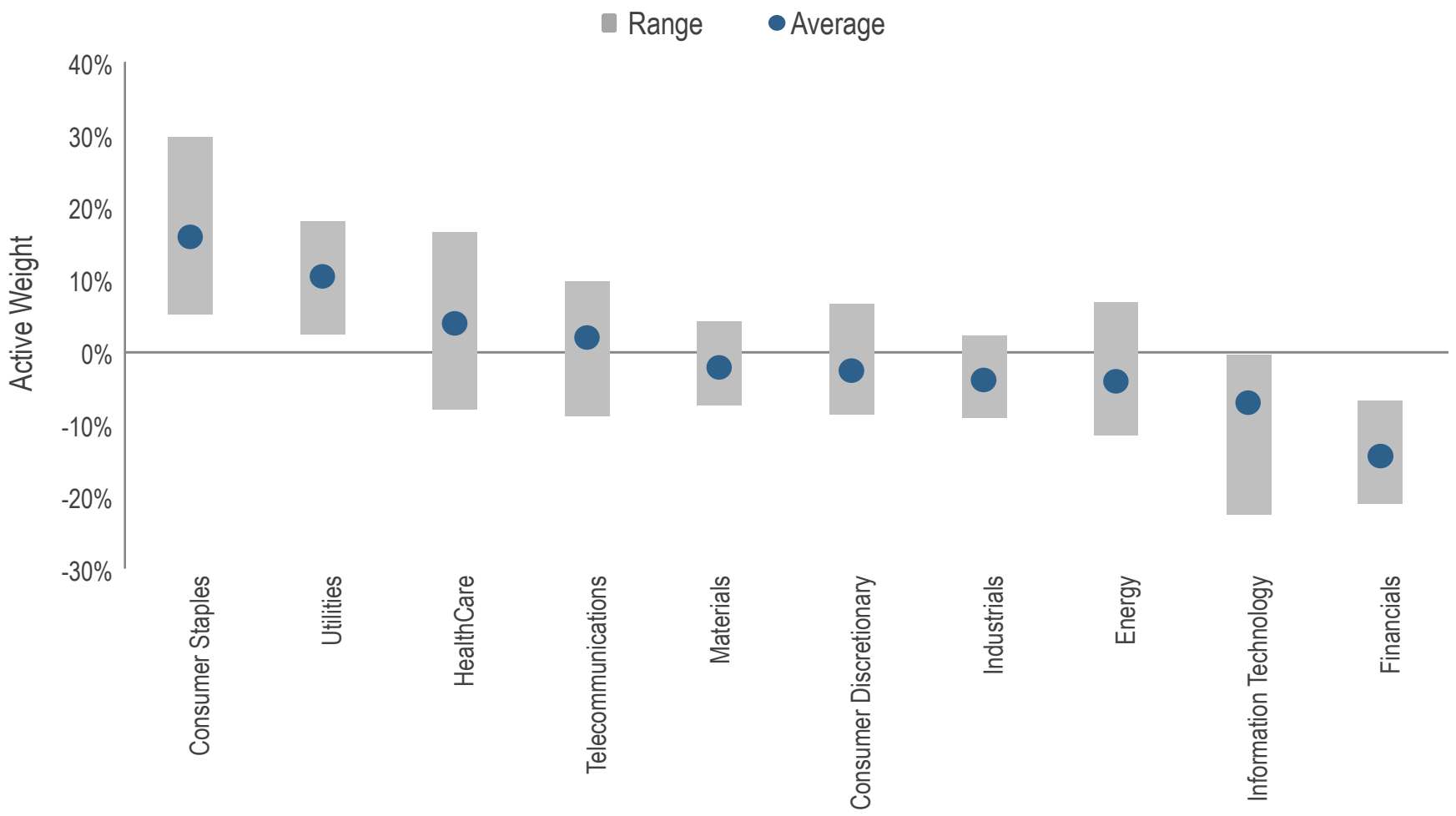
UNDERSTANDING MANAGED VOLATILITY: SECTOR EXPOSURES

1998 – 2013



Source: Acadian Asset Management LLC, AAM Universe of Securities
Methodology: Range of active sector weights for a hypothetical portfolio from October 1998 through September 2013.
For illustrative purposes only. This is meant to be an educational illustrative example of industry exposures and is not intended to represent exposures generated by an actual portfolio. The hypothetical results do not represent actual trading or an actual account, but were achieved by means of retroactive application of a model designed with the benefit of hindsight for the period specified above. Results reflect transaction costs and other implementation costs. Results do not reflect advisory fees or their potential impact. This hypothetical example is using portfolio construction techniques and parameters current as of the creation date for the most appropriate comparison to the other hypothetical examples presented within. Hypothetical results are not indicative of actual future results. Every investment program has the opportunity for loss as well as profit.

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OBSERVATIONS

A Sophisticated Portfolio Construction Process

- Incorporating security correlations may help reduce risk below simple volatility sorted portfolios

Return Forecasts

- Holding higher expected return stocks may increase returns without increasing risk

Careful Active Management May Add Value to Minimum Variance Portfolios

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APPLICATION

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MANAGED VOLATILITY LIES BETWEEN STOCKS AND BONDS

An equity strategy that has

- Returns similar to the aggregate market, with
- Lower risk
- Lower correlation with benchmark-tied strategies

…Can be used in many ways. For example, as a potential

- Risk reducer
- Return enhancer
- Hedge fund alternative
- Equity diversifier

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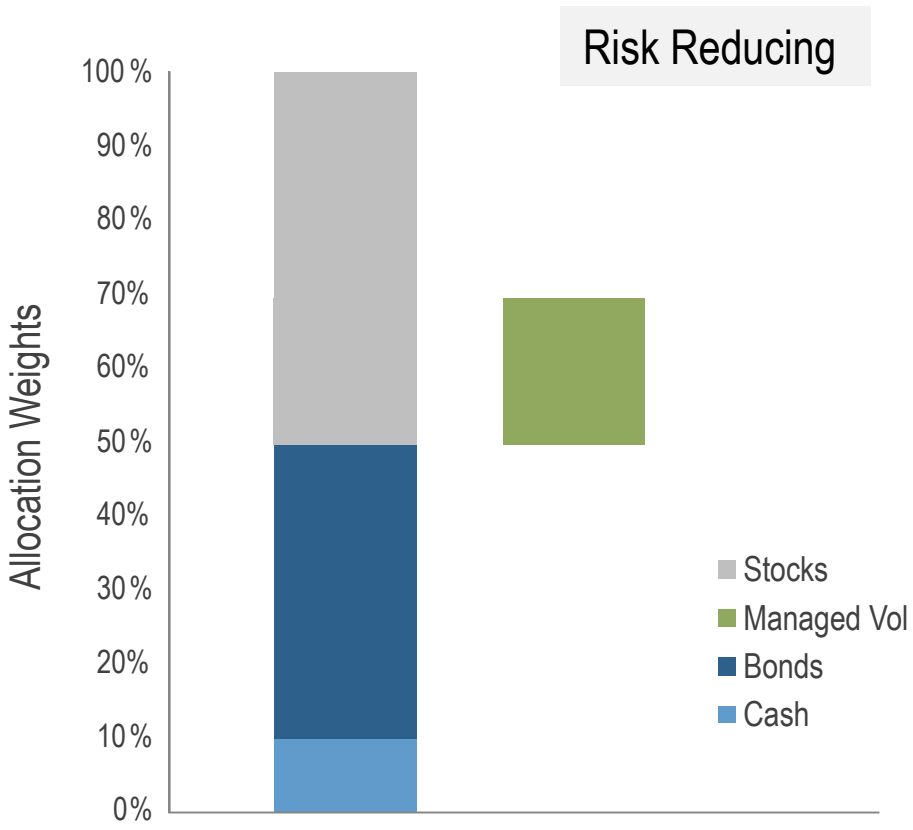
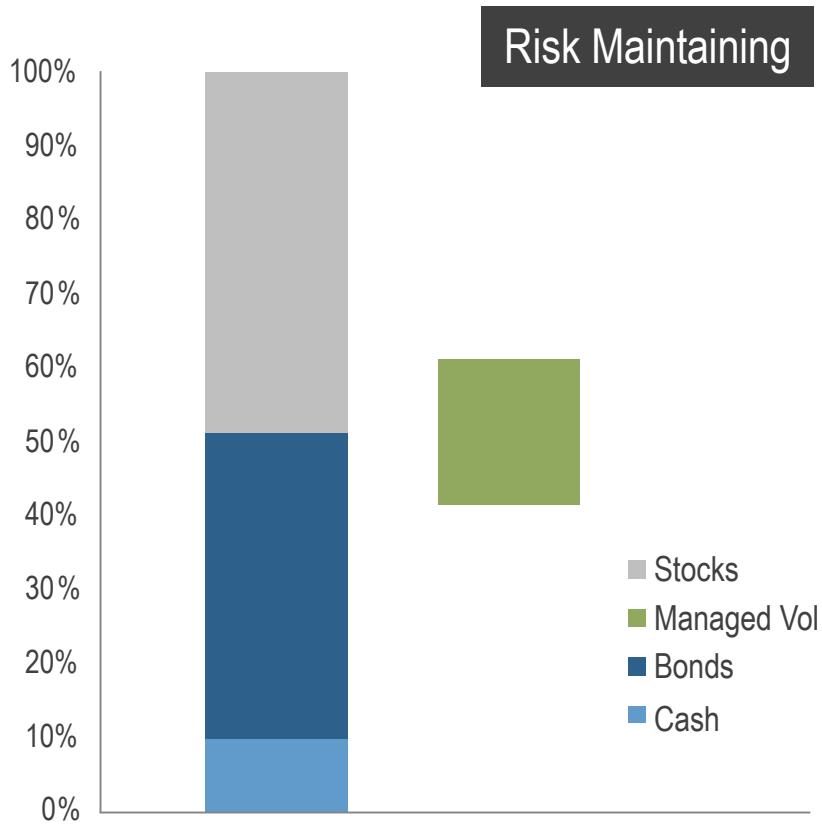
MANAGED VOLATILITY LIES BETWEEN STOCKS AND BONDS





- To reduce potential risk level, replace a portion of the stock allocation with a managed volatility portfolio

- To increase potential returns, replace a portion of both stock and bond allocations with a managed volatility portfolio

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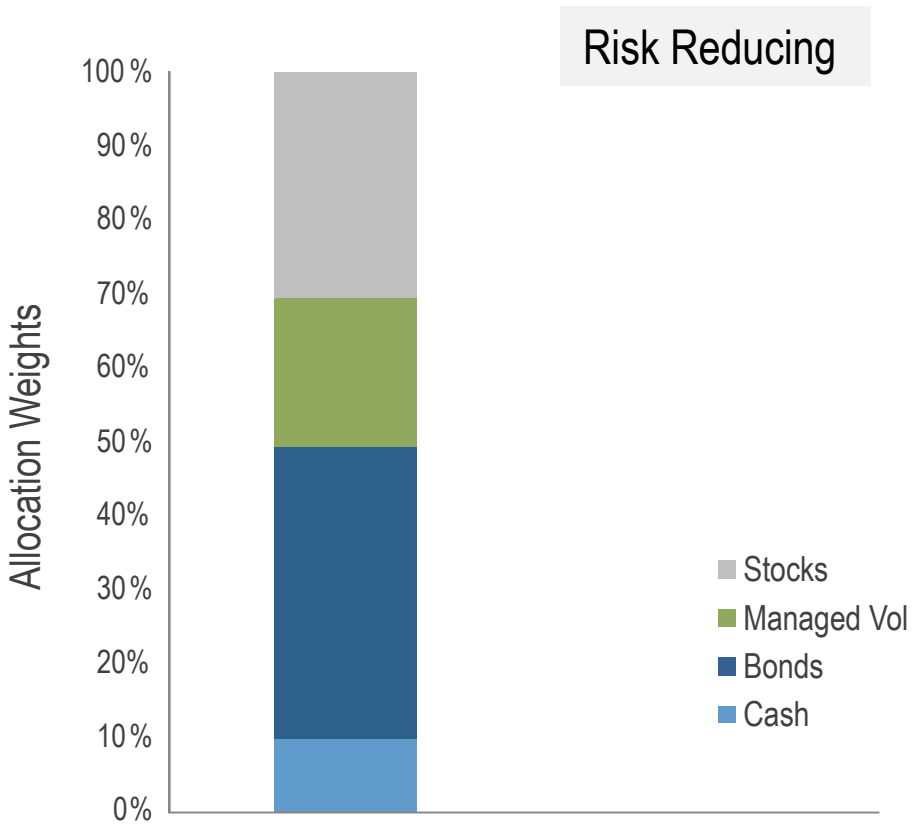
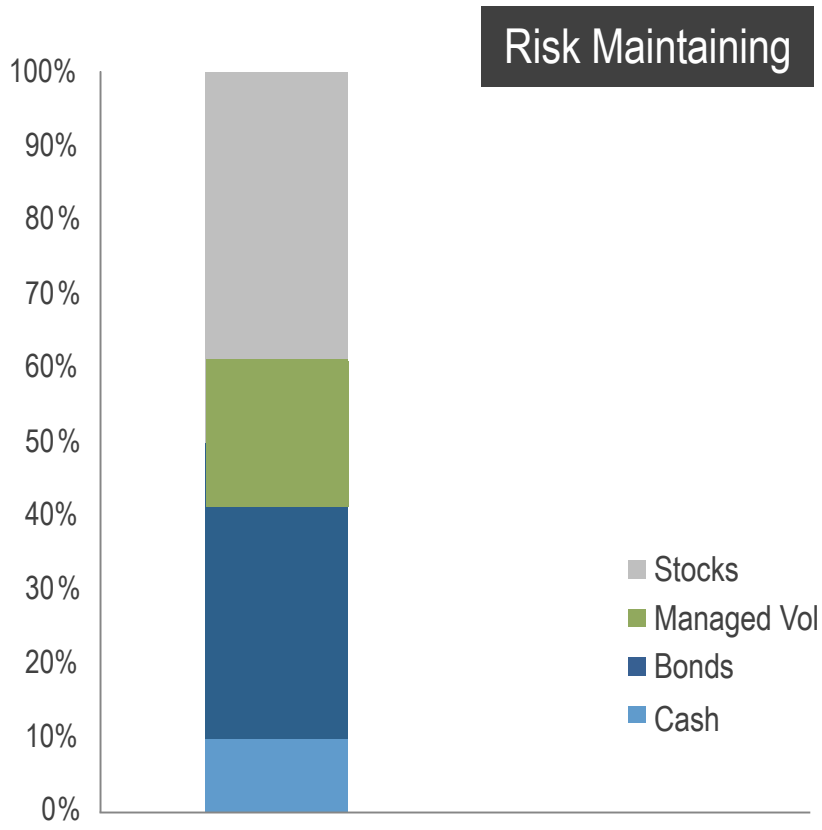
MANAGED VOLATILITY LIES BETWEEN STOCKS AND BONDS





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AN EXAMPLE OF ASSET ALLOCATION

ACROSS U.S. STOCKS, BONDS, AND CASH

Historical Risk, Return, and Correlation 1968 – 2012

	Annualized Returns (%)	Standard Deviation (%)	Stocks	Bonds	Cash	Minimum Variance
Stocks	9.5	15.5	1.00			
Bonds	8.4	10.3	0.14	1.00		
Cash	5.4	0.9	-0.01	0.04	1.00	
Minimum Variance	11.2	10.9	0.76	0.29	0.02	1.00

Risk Reducing Asset Allocation

Stocks	50%	40%	30%	20%	10%
Bonds	40%	40%	40%	40%	40%
Cash	10%	10%	10%	10%	10%
Minimum Variance*	0%	10%	20%	30%	40%
Annualized Return (%)	8.7	8.8	9.0	9.2	9.4
Annualized Risk (%)	9.3	8.7	8.3	8.0	7.8
Sharpe Ratio	0.36	0.40	0.44	0.48	0.51

Constant Risk Asset Allocation

Stocks	50%	47%	43%	38%	32%
Bonds	40%	33%	27%	22%	18%
Cash	10%	10%	10%	10%	10%
Minimum Variance*	0%	10%	20%	30%	40%
Annualized Return (%)	8.7	8.9	9.2	9.4	9.6
Annualized Risk (%)	9.3	9.3	9.3	9.3	9.3
Sharpe Ratio	0.36	0.38	0.40	0.43	0.45

*Hypothetical Portfolios. Source: Acadian Asset Management LLC, AAM US, CRSP, CRSP®, Center for Research in Security Prices. Graduate School of Business, The University of Chicago. Used with permission. All rights reserved. crsp.uchicago.edu, S&P Universe of Securities

In the above tables, stocks refer to the S&P 500 Index and bonds refer to the Barclays U.S. Government Aggregate Index. For illustrative purposes only. This is meant to be an educational illustrative example of asset allocation and is not intended to represent investment returns generated by an actual portfolio. The hypothetical results do not represent actual trading or an actual account, but were achieved by means of retroactive application of a model designed with the benefit of hindsight for the period specified above. Results are gross and would be reduced by advisory fees. Results reflect transaction costs and other implementation costs. Reference to the benchmark is for comparative purposes only. Hypothetical results are not indicative of actual future results. Every investment program has the opportunity for loss as well as profit. Index Source: Copyright © 2015, Standard & Poor's Financial Services LLC. All rights reserved.

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OBSERVATIONS

- The risk of managed volatility is between stocks and bonds
- Risk reducing versus risk maintaining approach

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CONCLUSION

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USE OF HYPOTHETICAL PERFORMANCE DISCLOSURE

Hypothetical performance results have many inherent limitations, some of which are described below. No representation is being made that any account will or is likely to achieve profits or losses similar to those shown. In fact, there are frequently sharp differences between hypothetical performance results and the actual performance results subsequently achieved by any particular trading program.

One of the limitations of hypothetical performance results is that they are generally prepared with the benefit of hindsight. In addition, hypothetical trading does not involve financial risk, and no hypothetical trading record can completely account for the impact of financial risk in actual trading. For example, the ability to withstand losses or to adhere to a particular trading program in spite of trading losses are material points which can also adversely affect actual trading results. There are numerous other factors related to the markets in general or to the implementation of any specific trading program which cannot be fully accounted for in the preparation of hypothetical performance results and all of which can adversely affect actual trading results.

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CONCLUSION

Low risk stocks have earned higher returns

- Around the world, over long time periods

Anomaly is likely to persist

- Investor preferences and expectations
- Standard benchmarks limit arbitrage

Managed volatility seeks to exploit the low-risk anomaly

- Intelligent portfolio construction process
- Potential for market returns with substantially lower risk

Managed volatility fits naturally within asset allocation

- In between equity and fixed income

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Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly

Malcolm Baker, Brendan Bradley, and Jeffrey Wurgler

Contrary to basic finance principles, high-beta and high-volatility stocks have long underperformed low-beta and low-volatility stocks. This anomaly may be partly explained by the fact that the typical institutional investor's mandate to beat a fixed benchmark discourages arbitrage activity in both high-alpha, low-beta stocks and low-alpha, high-beta stocks.

Among the many candidates for the greatest anomaly in finance, a particularly compelling one is the long-term success of low-volatility and low-beta stock portfolios. Over 1968–2008, low-volatility and low-beta portfolios offered an enviable combination of high average returns and small drawdowns. This outcome runs counter to the fundamental principle that risk is compensated with higher expected return. In our study, we applied principles of behavioral finance to shed light on the drivers of this anomalous performance and to assess the likelihood that it will persist.

Behavioral models of security prices combine two ingredients. The first is that some market participants are irrational in some particular way. In the context of the low-risk anomaly, we believe that a preference for lotteries and the well-established biases of representativeness and overconfidence lead to a demand for higher-volatility stocks that is not warranted by fundamentals.

The second ingredient is limits on arbitrage, which explain why the "smart money" does not offset the price impact of any irrational demand. With respect to the low-risk anomaly, we examined whether the underappreciated limit on arbitrage is benchmarking. Many institutional investors in a position to offset the irrational demand for risk have fixed-benchmark mandates (typically capitalization weighted), which, by their nature, discourage investments in low-volatility stocks. Drawing out the implications of Brennan's (1993) model of agency and asset prices, we looked at whether traditional fixed-benchmark mandates with a leverage constraint cause institutional investors to pass up the superior risk–return tradeoff of low-volatility portfolios; we also examined the appropriateness of a leverage constraint assumption. Rather than being a stabilizing force on prices, the typical institutional contract for delegated portfolio management could increase the demand for higher-beta investments.

Other researchers have attempted to explain the low-risk anomaly on the basis of behavioral elements. For example, Karceski (2002) pointed out that mutual fund investors tend to chase returns over time and across funds, possibly because of an extrapolation bias. These forces make fund managers care more about outperforming during bull markets than underperforming during bear markets, thus increasing their demand for high-beta stocks and reducing their required returns. In our study, we placed the irrationality elsewhere and focused on distortions introduced by benchmarking. Nevertheless, his model's predictions appear to complement our own, and the mechanisms could certainly work simultaneously.

The Low-Risk Anomaly

In an efficient market, investors realize above-average returns only by taking above-average risks. Risky stocks have high returns, on average, and safe stocks do not. This simple empirical proposition has been hard to support on the basis of the history of U.S. stock returns. The most widely used measures of risk point rather strongly in the *wrong* direction.

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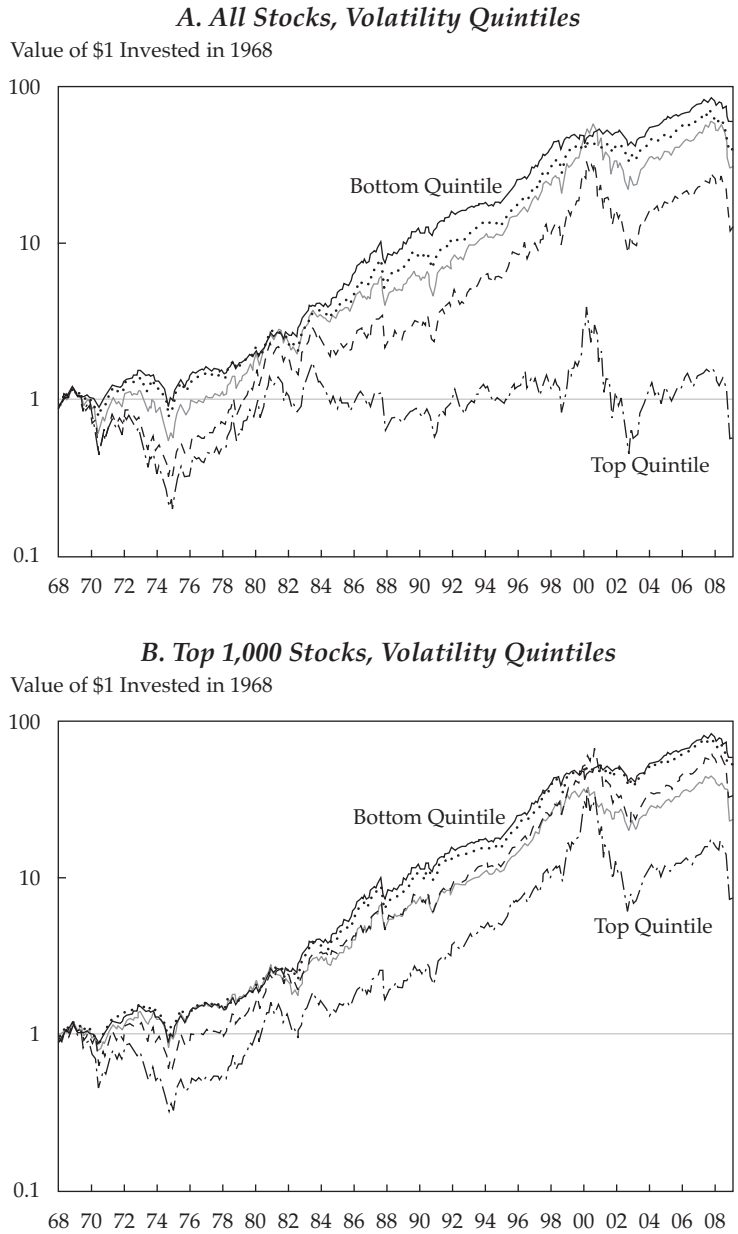
We obtained data for January 1968–December 2008 (a span of 41 years) from CRSP. We sorted stocks into five groups for each month according to either five-year trailing total volatility or trailing beta—thus using data going back to January 1963—and tracked the returns on these portfolios. We also restricted the investing universe to the top 1,000 stocks by market capitalization. **Figure 1** shows the results.

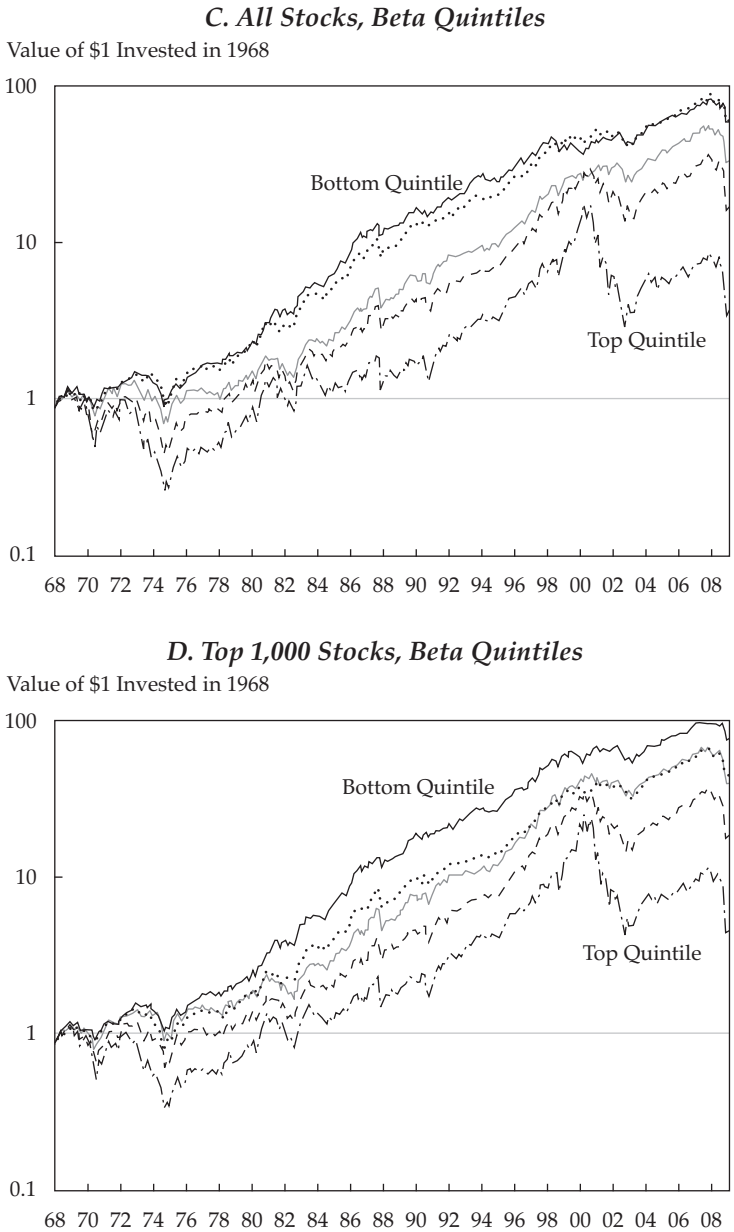
Regardless of whether we define risk as volatility or beta or whether we consider all stocks or only large caps, low risk consistently outperformed high risk over the period. Panel A shows that a dollar invested in the lowest-volatility portfolio in January 1968 increased to $59.55. Over this period, inflation eroded the real value of a dollar to about 17 cents, meaning that the low-risk portfolio produced a gain of $10.12 in real terms. Contrast this performance with that of the highest-volatility portfolio. A dollar invested there was worth 58 cents at the end of December 2008, assuming no transaction costs. Given the declining value of the dollar, the real value of the high-volatility portfolio declined to less than 10 cents—a 90 percent decline in real terms! Remarkably, an investor who aggressively pursued high-volatility stocks over the last four decades would have borne almost a total loss in real terms.

Figure 1. Returns by Volatility and Beta Quintile, January 1968–December 2008



(continued)

Figure 1. Returns by Volatility and Beta Quintile, January 1968–December 2008 (continued)



Notes: For each month, we sorted all publicly traded stocks (Panels A and C) and the top 1,000 stocks by market capitalization (Panels B and D) tracked by CRSP (with at least 24 months of return history) into five equal quintiles according to trailing volatility (standard deviation) and beta. In January 1968, $1 is invested according to capitalization weights. We estimated volatility and beta by using up to 60 months of trailing returns (i.e., return data starting as early as January 1963). At the end of each month, we rebalanced each portfolio, excluding all transaction costs.

Source: Acadian calculation with data from CRSP.

Panel C considers beta as the measure of risk. A dollar invested in the lowest-beta portfolio grew to $60.46 ($10.28 in real terms), and a dollar invested in the highest-beta portfolio grew to $3.77 (64 cents in real terms). Like the high-volatility investor, the high-beta investor failed to recover his dollar in real terms and underperformed his "conservative" beta neighbor by 964 percent.

Although almost all mispricings were stronger for small companies than for large companies, the low-risk anomaly was dramatic even for large companies. A dollar invested in low-volatility large caps grew to $53.81 over 41 years, whereas a dollar invested in high-volatility large caps grew to $7.35. For beta, the numbers are $78.66 and $4.70, respectively.

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Finally, as if this puzzle were not bad enough, other facts only compound it.

- The low-risk portfolios' paths to their higher dollar values have been much smoother than those of the high-risk portfolios. They are as advertised: genuinely lower risk.
- Motivated by the analysis of Pettengill, Sundaram, and Mathur (1995), we repeated the analysis separately for months in which market returns were above or below their median. Consistent with Pettengill et al. (1995), we found that high-beta stocks earned higher (lower) total returns than did low-beta stocks in up (down) markets, but on a capital asset pricing model (CAPM) market-adjusted basis, the low-beta anomaly was present in both environments. That low beta is high alpha is a robust historical pattern.
- The transaction costs of monthly rebalancing were substantially higher for the high-volatility portfolio, which means that the relative performance in Figure 1 is understated. We found similar results under yearly rebalancing; constraining turnover did not have a material effect.
- With the exception of the technology bubble, the return gap has, if anything, widened a bit since 1983—a period in which institutional investment managers became progressively more numerous, better capitalized, and more quantitatively sophisticated. Karceski (2002) also noted this trend.

These results are not new, but they have not been sufficiently emphasized, explained, or exploited. In the 1970s, Black (1972), Black, Jensen, and Scholes (1972), and Haugen and Heins (1975) noted that the relationship between risk and return was much flatter than predicted by the CAPM. Haugen and Heins pointed out that the relationship was not merely flat in their sample period but was actually inverted. Extending this analysis through 1990, Fama and French (1992) also found that the relationship was flat, prompting many to conclude that beta was dead. More recently, Ang, Hodrick, Xing, and Zhang (2006, 2009) drew renewed attention to these results, finding that high-risk stocks have had "abysmally low average returns" (2006, p. 296) in longer U.S. samples and in international markets. Blitz and van Vliet (2007) provided a detailed analysis of the volatility anomaly and demonstrated its robustness across regions and to controls for size, value, and momentum effects. Frazzini and Pedersen (2010) documented that low-risk securities have high risk-adjusted returns in global stock, Treasury, credit, and futures markets. Bali, Cakici, and Whitelaw (forthcoming) investigated a measure of lottery-like return distributions, which is highly correlated with other risk measures, and found that it too is associated with poor performance. All told, the evidence for a risk–return trade-off along the lines of the CAPM has, if anything, only deteriorated in the last few decades.

These patterns are hard to explain with traditional, rational theories of asset prices. In principle, beta might simply be the wrong measure of risk. The CAPM is just one equilibrium model of risk and return, with clearly unrealistic assumptions. For the past few decades, finance academics have devoted considerable energy to developing rational models, searching for the "right" measure of risk. Most of these newer models make the mathematics of the CAPM look quaint.

Despite superior computational firepower, however, the new models face an uphill battle. After all, the task is to prove that high-volatility and high-beta stocks are *less* risky. A less risky stock might not be less volatile (although volatility and beta are positively correlated in the cross section), but it must at least provide insurance against bad events. Even this notion of risk fails to resolve the anomaly. The high-volatility-quintile portfolio provided a relatively low return in precisely those periods when an insurance payment would have been most welcome, such as the downturns of 1973–1974 and 2000–2002, the crash of 1987, and the financial crisis that began in the fall of 2008. Investors appeared to be paying an insurance premium only to lose even more whenever the equity market (and often the economy) underwent a meltdown.

We believe that the long-term outperformance of low-risk portfolios is perhaps the greatest anomaly in finance. Large in magnitude, it challenges the basic notion of a risk–return trade-off.

A Behavioral Explanation

In our study, we hypothesized two drivers of these results: (1) less than fully rational investor behavior and (2) underappreciated limits on arbitrage. The combination of these two forces is the basic framework of behavioral asset pricing, as laid out in such surveys as Shleifer (2000), Barberis and Thaler (2003), and Baker and Wurgler (2007). We explored a new combination that could explain the low-risk anomaly.

The Irrational Preference for High Volatility. The preference for high-volatility stocks derives from the biases that afflict the individual investor. We examined three such biases.

■ *Preference for lotteries.* Would you take a gamble with a 50 percent chance of losing $100 versus a 50 percent chance of winning $110? Most people would say no. Despite the positive expected payoff, the possibility of losing $100 is enough to deter participation, even when $100 is trivial compared with wealth or income.

Kahneman and Tversky (1979) called this behavior "loss aversion." Taken on its own, loss aversion suggests that investors would shy away from volatility for fear of realizing a loss. But something strange happens as the probabilities shift. Now suppose that you are offered a gamble with a near-certain chance of losing $1 and a small (0.12 percent) chance of winning $5,000. As in the first example, this gamble has a positive expected payoff of around $5. In this case, however, most people take the gamble. Gambling on lotteries and roulette wheels, which have negative expected payoffs, is a manifestation of this tendency.

To be precise, this behavior is more about positive skewness, whereby large positive payoffs are more likely than large negative ones, than it is about volatility. But Mitton and Vorkink (2007) reminded us that volatile individual stocks, with limited liability, are also positively skewed. Buying a low-priced, volatile stock is like buying a lottery ticket: There is a small chance of its doubling or tripling in value in a short period and a much larger chance of its declining in value. Boyer, Mitton, and Vorkink (2010) argued that volatility is a proxy for expected skewness. Kumar (2009) found that some individual investors do show a clear preference for stocks with lottery-like payoffs, measured as idiosyncratic volatility or skewness. Modeling this preference with the cumulative prospect theory approach in Tversky and Kahneman (1992), Barberis and Huang (2008) examined an array of circumstantial evidence that volatile stocks are overvalued because of a lottery preference. Blitz and van Vliet (2007) tied the preference for lottery tickets to the behavioral portfolio theory in Shefrin and Statman (2000). Barberis and Xiong (2010) offered a preference-based explanation of the volatility effect that is separate from a skewness preference in which investors obtain utility from *realizing* gains and losses on risky assets, not from paper gains and losses.

■ *Representativeness.* The classic way to explain representativeness is with an experiment from Tversky and Kahneman (1983). They described a fictional woman named Linda as "single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations" (p. 297). They then asked subjects which was more probable: A (Linda is a bank teller) or B (Linda is a bank teller who is active in the women's movement). The fact that many subjects chose B shows that probability theory and Bayes' rule are not ingrained skills. As a proper subset of A, B is less likely than A but seems more "representative" of Linda.

What does this experiment have to do with stocks and volatility? Consider defining the characteristics of "great investments." The layman and the quant address this question with two different approaches. On the one hand, the layman, trying to think of great investments—perhaps buying Microsoft Corporation and Genzyme Corporation at their IPOs in 1986—concludes that the road to riches is paved with speculative investments in new technologies. The problem with this logic is similar to the Linda question. Largely ignoring the high base rate at which small, speculative investments fail, the layman is inclined to overpay for volatile stocks.

The quant, on the other hand, analyzes the full sample of such stocks as Microsoft and Genzyme, as shown in Figure 1. She concludes that without a way to separate the Microsofts from the losers, high-risk stocks are generally to be avoided.

■ *Overconfidence.* Another pervasive bias underlying the preference for high-volatility stocks is overconfidence (see Fischhoff, Slovic, and Lichtenstein 1977; Alpert and Raiffa 1982). For example, experimenters ask subjects to estimate the population of Massachusetts and to provide a 90 percent confidence interval around their answer. Most people form confidence intervals that are too narrow. And the more obscure the question—Bhutan instead of Massachusetts—the more this calibration deteriorates.

Valuing stocks involves the same sort of forecasting. What will revenues be five years hence? Overconfident investors are likely to disagree. Being overconfident, they will also agree to disagree, sticking with the false precision of their estimates. The extent of disagreement is likely higher for more uncertain outcomes—such as the returns on high-volatility stocks. Cornell (2009) viewed overconfidence as an important part of the demand for volatile stocks.

The careful theorist will note that one extra assumption is needed to connect overconfidence—or, more generally, differences of opinion—to the demand for volatile stocks. In markets, pessimists must act less aggressively than optimists. Investors must have a general reluctance or inability to short stocks relative to buying them. Empirically, the relative scarcity of short sales among individual investors and even institutional investors is evident, so

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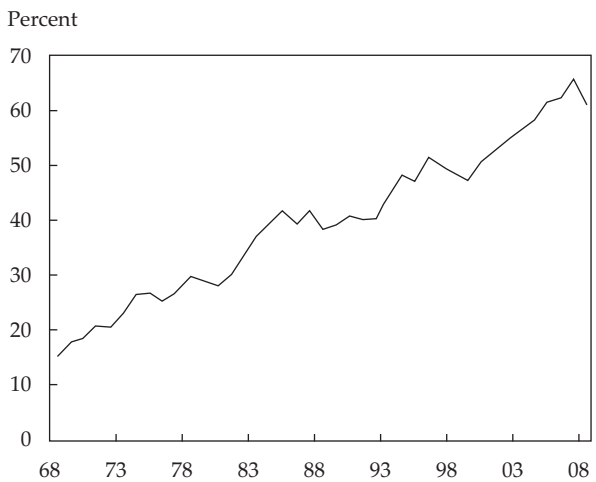
this assumption is clearly valid. It means that prices are generally set by optimists, as pointed out by Miller (1977). Stocks with a wide range of opinions will have more optimists among their shareholders and will sell for higher prices, leading to lower future returns. Diether, Malloy, and Scherbina (2002) provided empirical support for this idea.

Benchmarking as a Limit on Arbitrage. Assuming that average investors have a psychological demand for high-volatility stocks, the remaining and deeper economic question is why sophisticated institutions do not capitalize on the low-risk/high-return anomaly. Indeed, as mentioned earlier, this anomaly gained force over a period when institutional management in the United States went from 30 percent to 60 percent (**Figure 2**).

Figure 2. Institutional Ownership, 1968–2008



Note: This figure depicts data on institutional ownership from the Federal Reserve's Flow of Funds Accounts of the United States, Table L.213: assets managed by insurance companies (lines 12 and 13), public and private pension funds (lines 14, 15, and 16), open- and closed-end mutual funds (lines 17 and 18), and broker/dealers (line 20); assets under management are scaled by the market value of domestic corporations (line 21).

One issue is why institutional investors do not short the very poor-performing top volatility quintile. For the full CRSP sample, this question has a simple answer: The top volatility quintile tends to be small stocks, which are costly to trade in large quantities—both long and, especially, short—the volume of shares available to borrow is limited, and borrowing costs are often high. In the large-cap sample, the same frictions are present, albeit in considerably smaller measure, which begs the second and more interesting question: Why do institutional investors not at least overweight the low-volatility quintile? We believe that the answer involves benchmarking.

A typical contract for institutional equity management contains an implicit or explicit mandate to maximize the "information ratio" relative to a specific, fixed capitalization-weighted benchmark without using leverage. For example, if the benchmark is the S&P 500 Index, the numerator of the information ratio (IR) is the expected difference between the return earned by the investment manager and the return on the S&P 500. The denominator is the volatility of this return difference, also called the tracking error. The investment manager is expected to maximize this IR through stock selection and without using leverage. Sensoy (2009) reported that 61.3 percent of U.S. mutual fund assets are benchmarked to the S&P 500 and 94.6 percent are benchmarked to some popular U.S. index. Under current U.S. SEC rules, all mutual funds must select a benchmark and show fund returns versus the benchmark in their prospectuses. In this segment of the asset management industry, however, the use of the IR is less formalized.

This contract is widely used because it has several appealing features. Although the ultimate investor cares more about total risk than tracking error, it is arguably easier to understand the skill of an investment manager—and the risks taken—by comparing returns with those of a well-known benchmark. Knowing that each manager will at least roughly stick to a benchmark also helps the ultimate investor keep track of the overall risk across many asset classes and mandates.

But these advantages come at a cost. Roll (1992) analyzed the distortions that arise from a fixed-benchmark mandate, and Brennan (1993) considered the effect on stock prices. Cornell and Roll (2005) developed a similar model. In particular, *a benchmark makes institutional investment managers less likely to exploit the low-volatility anomaly*. We lay this model out formally in Appendix A, but the logic is simple.

In the Sharpe–Lintner CAPM, investors with common beliefs aim to maximize the expected return on their portfolios and minimize volatility. This goal leads to the famously simple relationship between beta risk and return. A stock's expected return equals the risk-free rate plus its beta times the market risk premium:

$$E(R) = R_f + \beta E\left(R_m - R_f\right). \tag{1}$$

Now imagine some extra demand for high-volatility stocks. This demand will push up the price of higher-risk stocks and drive down their expected returns, and vice versa for lower-risk stocks.

Problem Cases: Low Beta/High Alpha and Low Alpha/High Beta

An institutional investor with a fixed benchmark is surprisingly unlikely to exploit such mispricings. In fact, in empirically relevant cases, the manager's incentive is to exacerbate them.

Low Beta/High Alpha. Consider an institutional manager who is benchmarked against the market portfolio. Suppose that the expected return on the market is 10 percent more than the risk-free rate and the volatility of the market is 20 percent. Take a stock with a β of 0.75 and imagine that it is undervalued, with an expected return greater than the CAPM benchmark in Equation 1 by an amount α. Overweighting the stock by a small amount—say, 0.1 percent—will increase the expected active, or benchmark-adjusted, return by approximately $0.1\%\left\{\left[\alpha-(1-\beta)\right]\left[E\left(R_m-R_f\right)\right]\right\}=0.1\%(\alpha-2.5\%)$. The extra tracking error of the portfolio is at least $\sqrt{0.1\%^2\left[\sigma_m^2(1-\beta)^2\right]}=\sqrt{0.1\%^2(0.0025)}$, the component that comes from having a portfolio β that is not equal to 1.

This investment manager will not start overweighting such an undervalued low-beta stock until its α exceeds 2.5 percent a year. An undervalued stock with a less extreme but still substantial alpha—say, 2 percent—is actually a better candidate for *underweighting*.

A key assumption here is that the manager cannot use leverage. By borrowing 33 percent of each dollar invested in the low-beta stock, the problem of portfolio tracking error is solved, at least with respect to the β component. Black (1972) also noted the relevance of a leverage constraint to a flat return–beta relationship. Similarly, a balanced fund mandate without a fixed-leverage constraint could solve this problem. For example, if a balanced fund mandate dictated a beta of 0.5 rather than a fixed 50 percent of the portfolio in stocks, the manager could choose low-risk stocks in place of a greater percentage of the portfolio in low-beta fixed-income securities. There are also more elaborate solutions to the problem of delegated investment management (e.g., van Binsbergen, Brandt, and Koijen 2008).

Therefore, our assumption of a leverage constraint deserves consideration. We believe that it is a reasonable assumption for a large portion of the asset management industry. Although precise statistics are hard to come by, conventional wisdom says that few mutual funds use leverage. We spot-checked and found that the five largest active domestic equity mutual funds did not use any leverage as of 1 July 2010. The Investment Company Act of 1940 prohibits mutual funds from using more than 33 percent leverage; our example assumes that funds use this statutory maximum.

Using data on U.S. holdings from LionShares and CRSP betas, we found that mutual funds with *balanced* in their names had an average equity beta of 1.02 in December 2008. This number is slightly lower than the average beta of other mutual funds (1.10) but is still above 1. (Using older data [1984–1996], Karceski [2002] reported an average beta of 1.05.) In addition, the assets under management of balanced funds were only 2 percent of the total.

Closed-end funds use leverage, but they are a small portion of total assets under management. Moreover, Anand (2009) reported that only 2 funds (out of the 18 he considered) used substantial leverage and had assets greater than $100 million. He also found that most of the funds used a 130/30 strategy, which typically involves leverage through borrowed stock, not bonds, and so the same benchmarking challenges arise in attempting to exploit the low-volatility anomaly. We are unaware of any comprehensive tabulation of institutional mandates—in either benchmarking or leverage.

Certainly, some strategies allow the flexibility to take advantage of the anomaly without running into the benchmark limit on arbitrage. These include maximum Sharpe ratio, managed volatility, balanced, and a variety of hedge fund strategies. Although data on the total assets managed under these strategies are unavailable, anecdotal evidence suggests that the numbers are relatively modest. Even when the explicit contract allows flexibility, investment managers do not overweight low-risk stocks. One possible interpretation is that balanced funds, for example, are implicitly evaluated according to their allocation to equities, not their beta. As mentioned earlier, Sensoy (2009) reported that almost all actively managed U.S. equity mutual funds are benchmarked to an S&P or Russell index.

In the end, our model is a substantial simplification of an enormously heterogeneous market, but the assumption of a leverage constraint seems likely to be a reasonable approximation. The documentable amount of assets under management that use leverage or that can use leverage is small relative to the market caps of the stocks involved in the anomaly—which, in some sense, is the total capitalization of the stock market—and is small relative to the amount of capital that would be required not only to *flatten* the relationship between risk and expected return but also to *reverse* it, as traditional finance theory would prefer.

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Low Alpha/High Beta. Now consider the case of overvalued high-beta stocks. By the same logic, the manager will not underweight a stock with a β of 1.25, for example, until its α is below –2.5 percent. And again, the manager becomes part of the problem unless the alpha is very negative—an α of –2 percent, for example, is still a candidate for *overweighting*.

The logic illustrates that an investment manager with a fixed benchmark and no leverage is best suited to exploit mispricings among stocks with close to market risk (i.e., a β near 1). In those cases, managers will have a robust desire to overweight positive-alpha stocks and underweight negative-alpha stocks, thus enhancing market efficiency. As beta decreases (increases), alpha must increase (decrease) to induce bets in that direction. All of this relates directly to the low-risk anomaly, whose essence is that low risk is undervalued relative to high risk. This finding is not surprising in a benchmarked world.

Table 1 gives a feel for just what these anomalies look like to the benchmarked manager. Let us focus on the case of large caps only, a universe of special practical relevance to benchmarked investors and a perfectly dramatic illustration of the problem. We assume that the benchmark is the CRSP value-weighted market return for the three major U.S. exchanges. For low-volatility portfolios, the Sharpe ratio is reasonably high, at 0.38. But the IR—the ratio of the excess return over the fixed benchmark to the tracking error—is much less impressive, at 0.08. The results for the other three low-risk portfolios offer a similar message.

Beta and volatility are highly correlated. Determining which notion of risk is more fundamental to the anomaly is of practical interest. It is also of theoretical interest because our mechanism centers on beta, with total volatility entering the picture only to the extent that portfolios are not sufficiently diversified to prevent idiosyncratic risk from affecting tracking error. In unreported results, we sorted on volatility orthogonalized to beta (roughly, idiosyncratic risk) and on beta orthogonalized to volatility.

The results suggest that beta is closer than volatility to the heart of the anomaly. For large-cap stocks, high-orthogonalized-beta portfolios have the lowest returns, just as high-raw-beta portfolios do. But large stocks with high orthogonalized volatility actually show higher returns. In other words, beta drives the anomaly in large stocks, but both measures of risk play a role in small stocks. This pattern is consistent with the fact that benchmarked managers focus disproportionately on large stocks.

The main point of Table 1 is that a benchmarked institutional fund manager is likely to devote little long capital or risk-bearing capacity to exploiting these risk anomalies. Nor is aggressively shorting high-risk stocks a particularly appealing strategy. Other anomalies generated far better IRs over this period. Using data from Ken French's website,[1] we found (in unreported results) an IR of 0.51 for a simple, long-only, top quintile value strategy over the same period (1968–2008); the IR of a simple, long-only, top quintile momentum strategy over the same period was 0.64. These vanilla quantitative strategy IRs suggest that the IR of a long-only strategy in low-volatility stocks is unappealing at 0.08 to 0.17. It will not draw much risk capital, and thus, the mispricings are likely to survive.

We can also think of all this in familiar CAPM terms. In a simple equilibrium described in Appendix A along the lines of Brennan (1993), with no irrational investors at all, the presence of delegated investment management with a fixed benchmark will cause the CAPM relationship to fail. In particular, it will be too flat, as shown in **Figure 3**:

$$E(R) = (R_f + c) + \beta E(R_m - R_f - c). \tag{2}$$

The constant, $c > 0$, depends intuitively on the tracking error mandate of the investment manager (a looser mandate leads to more distortion) and on the fraction of asset management that is delegated (more assets increase distortion). The pathological regions are the areas between the CAPM and the delegated management security market lines. For stocks in these regions, the manager will not enforce the CAPM and will be reluctant to overweight low-beta, high-alpha stocks and to underweight high-beta, low-alpha stocks. This finding is consistent with the average mutual fund beta of 1.10 over the last 10 years.

The presence of volatility-preferring, irrational investors serves only to further diminish the risk–return trade-off. As a theoretical matter, behavioral biases are not needed for benchmarking to flatten the CAPM relationship. We included this element in the discussion because behavioral biases are a fact, and so including them allows for a more accurate description of the phenomenon. In addition, we would be the first to acknowledge that benchmarking per se is unlikely to generate the full magnitude of the low-risk anomaly, whereby the risk–return relationship is inverted. Time variation in the strength of behavioral biases, attached to bubbles and crashes, may also help explain some of the time variation in the returns to risky versus safe securities; Baker and Wurgler (2007) studied fluctuations in investor sentiment.

Table 1. Returns by Volatility and Beta Quintile, January 1968–December 2008

	All Stocks					Top 1,000 Stocks				
	Low	2	3	4	High	Low	2	3	4	High
A. Volatility sorts										
Geometric average $R_p - R_f$	4.38%	3.37%	2.72%	0.46%	–6.78%	4.12%	4.03%	2.06%	2.81%	–0.82%
Average $R_p - R_f$	5.15%	4.75%	5.04%	4.18%	–1.73%	4.86%	5.12%	3.60%	5.02%	2.95%
Standard deviation	13.10%	16.72%	21.38%	26.98%	32.00%	12.74%	15.15%	17.48%	20.86%	27.13%
Sharpe ratio	0.39	0.28	0.24	0.16	–0.05	0.38	0.34	0.21	0.24	0.11
Average $R_p - R_m$	1.05%	0.65%	0.94%	0.08%	–5.84%	0.62%	0.88%	–0.64%	0.78%	–1.29%
Tracking error	6.76%	4.59%	7.88%	14.23%	20.33%	7.45%	5.54%	4.53%	7.91%	14.95%
Information ratio	0.16	0.14	0.12	0.01	–0.29	0.08	0.16	–0.14	0.10	–0.09
Beta	0.75	1.01	1.28	1.54	1.71	0.70	0.88	1.06	1.24	1.54
Alpha	2.08%	0.61%	–0.21%	–2.12%	–8.73%	2.00%	1.49%	–0.76%	–0.07%	–3.36%
t(Alpha)	2.44	0.85	–0.21	–1.19	–3.28	2.03	1.70	–1.07	–0.07	–1.84
B. Beta sorts										
Geometric average $R_p - R_f$	4.42%	4.49%	2.99%	1.27%	–2.42%	5.09%	3.75%	3.44%	1.46%	–1.89%
Average $R_p - R_f$	5.07%	5.30%	4.30%	3.36%	1.53%	5.74%	4.69%	4.72%	3.35%	1.56%
Standard deviation	12.13%	13.39%	16.31%	20.24%	27.77%	12.40%	14.07%	16.24%	19.27%	25.95%
Sharpe ratio	0.42	0.40	0.26	0.17	0.05	0.46	0.33	0.29	0.17	0.06
Average $R_p - R_m$	0.97%	1.20%	0.20%	–0.74%	–2.58%	1.50%	0.45%	0.48%	–0.89%	–2.68%
Tracking error	9.74%	7.06%	5.15%	6.25%	14.52%	8.83%	6.13%	4.31%	5.70%	13.02%
Information ratio	0.10	0.17	0.04	–0.12	–0.18	0.17	0.07	0.11	–0.16	–0.21
Beta	0.60	0.76	0.97	1.23	1.61	0.63	0.81	0.98	1.18	1.52
Alpha	2.60%	2.20%	0.31%	–1.69%	–5.06%	3.16%	1.38%	0.70%	–1.47%	–4.66%
t(Alpha)	2.23	2.39	0.39	–2.13	–2.97	2.77	1.53	0.99	–2.02	–3.15

Notes: For each month, we formed portfolios by sorting all publicly traded stocks (first five columns) and the top 1,000 stocks by market capitalization (second five columns) tracked by CRSP into five equal-sized quintiles according to trailing volatility (standard deviation) for Panel A and trailing beta for Panel B. We estimated volatility and beta by using up to 60 months of trailing returns (i.e., return data starting as early as January 1963). The return on the market, R_m, and the risk-free rate, R_f, are from Ken French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The information ratio uses the market return for the relevant universe, all stocks in the first five columns and the top 1,000 stocks in the last five columns. Average returns are monthly averages multiplied by 12. Standard deviation and tracking error are monthly standard deviations multiplied by the square root of 12.

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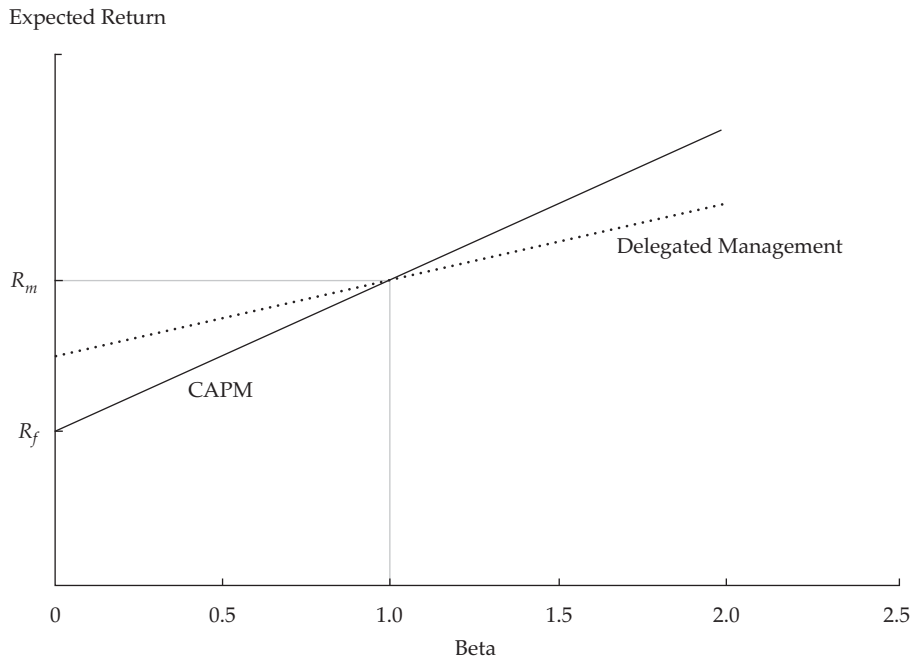
Figure 3. Delegated Investment Management with a Fixed Benchmark: R_m Flattens the CAPM Relationship



Putting the Pieces Together

To summarize, the combination of irrational investor demand for high volatility and delegated investment management with fixed benchmarks and no leverage flattens the relationship between risk and return. Indeed, the empirical results suggest that, over the long haul, the risk–return relationship has not merely been flattened but inverted. Yet sophisticated investors are, to a large extent, sidelined by their mandates to maximize active returns subject to benchmark tracking error.

Unfortunately, conducting a direct test of our proposed mechanism is difficult. Instead, we have presented evidence that is consistent with it. We consider the process-of-elimination findings of Ang et al. (2009), who ruled out several potential explanations, to be supportive evidence. To our knowledge, the most direct evidence for our mechanism is provided in Brennan and Li (2008), following up on the framework of Brennan (1993), which we also used. Brennan and Li found evidence that beta with the idiosyncratic component of the S&P 500 should have a negative payoff, all else being equal, consistent with investment managers' attempting to minimize tracking error by holding such stocks. Brennan and Li did not connect their results to the low-risk anomaly.

Another testable prediction is that as benchmarking has increased, the low-risk anomaly should likewise have become more severe. In unreported results, we found that this prediction is directionally correct and, depending on the sample, marginally statistically significant. For example, for the top 500 capitalization stocks in the full CRSP sample (1931–2008), we found that the relative return of low-minus-high volatility may have increased by 1 or 2 bps a year. But this test is not a powerful one. Return data are quite noisy, partly because the preference for volatility has varied with time. For example, the internet bubble, which focused on high-volatility stocks, would have swamped the effect of benchmarks as a limit on arbitrage over that period. We hope that future research will develop more powerful tests of our proposed mechanism.

From a practitioner perspective, the takeaway is that there is a solid investment thesis for low-volatility (and low-beta) strategies. If our explanation is valid, this thesis will be the case so long as fixed-benchmark contracts remain pervasive and the share of the market held by investment managers remains high. There is no reason to expect that the anomaly will go away any time soon.

Within vs. Across Mandates

There is an additional, more subtle prediction that we can test empirically. Investment managers with fixed benchmarks may not exploit mispricings when stocks of different risks have similar returns *within a particular mandate*. But risk and return are likely to line up *across mandates* if the ultimate

investors are thoughtful about asset allocation—for example, between intermediate and long-term bonds, between government and corporate bonds, between stocks and bonds, and between large-cap and small-cap stocks.

Table 2 shows that the CAPM does indeed work, to some extent, across asset classes, in contrast to its long-term performance within the stock market. In other words, as the β rises from 0.05 for intermediate-term government bonds to 1.07 for small-company stocks, average returns rise from 7.9 percent to 13.0 percent. A small CAPM anomaly in bonds still exists, whereby lower-risk asset classes appear to outperform their risk-adjusted benchmarks, which also suggests a possible impact from fixed benchmarks in tactical asset allocation. Although the returns on small-company stocks appear to be an exception, note that these returns are on lower-beta small stocks. Higher-beta small stocks have underperformed.

Searching for Lower Volatility

One last notable feature of both Figure 1 and Table 1 is compounding. The advantage of a low-risk portfolio versus a high-volatility portfolio is greater when displayed in compound returns than in average returns. The difference comes from the benefits of compounding a lower-volatility monthly series.

Given the power of compounding low-volatility returns and the outperformance of low-volatility stocks, a natural question is whether we can do even better than the low-risk-quintile portfolios by taking further advantage of the benefits of diversification. Returns aside, we can do better if we have useful estimates of not only individual company volatility but also the correlations among stocks. A portfolio of two *uncorrelated* but slightly more individually volatile stocks can be even less volatile than a portfolio of two *correlated* stocks with low volatility.

With that in mind, we constructed two minimum-variance portfolios that took advantage of finer detail in the covariance matrix. Following the method of Clarke, de Silva, and Thorley (2006), we used only large caps and a simple five-factor risk model—a realistic and implementable strategy—and compared the returns on two optimized low-volatility portfolios with the performance of the lowest quintile sorted by volatility (**Table 3**). In the second column of Table 3, we used individual company estimates of volatility, rather than a simple sort, to form a low-volatility portfolio but set the correlations among stocks to zero. In the third column, we also used the covariance terms from the risk model. We were able to reduce the total volatility of the portfolios from 12.7 percent with a simple sort to 11.5 percent in the optimized portfolio. This volatility reduction comes entirely from the estimation of correlations because the diagonal covariance model produces a higher-risk portfolio than does the simple sort. Moreover, because the reduction in volatility comes at no expense in terms of average returns, the Sharpe ratios are best in the optimized portfolio, as is visually apparent in **Figure 4**. These patterns are stable across both halves of our 41-year sample period.

The final column of Table 3 concerns leverage. With the leverage constraint relaxed, the high Sharpe ratio of the low-volatility portfolio in the third column can be converted into a respectable IR of 0.45. With leverage to neutralize the portfolio beta, the extra tracking error that comes from focusing on lower-beta stocks is reduced to the idiosyncratic component of stock selection. This portfolio produces higher-than-market returns at market levels of average risk.

Table 2. Risk and Return across Asset Classes, January 1968–December 2008

		Sharpe Ratio			CAPM Performance		
	Average Return	Excess Return	Std. Dev.	Sharpe	Beta	Alpha	t(Alpha)
Short government bonds	5.70%	0.00%	0.02%				
Intermediate government bonds	7.88	2.18	5.58	0.39	0.05	1.98%	2.28
Long government bonds	8.90	3.20	10.54	0.30	0.14	2.61	1.61
Corporate bonds	8.48	2.77	9.58	0.29	0.18	2.01	1.40
Large-company stocks	9.86	4.16	15.33	0.27	1.00		
Small-company stocks	13.04	7.34	21.79	0.34	1.07	2.88	1.28

Notes: Using data from Ibbotson Associates, we computed the average return and beta by asset class. The return on the market, R_m (large-company stocks), and the risk-free rate, R_f, are from Ibbotson Associates. Average returns are monthly averages multiplied by 12.

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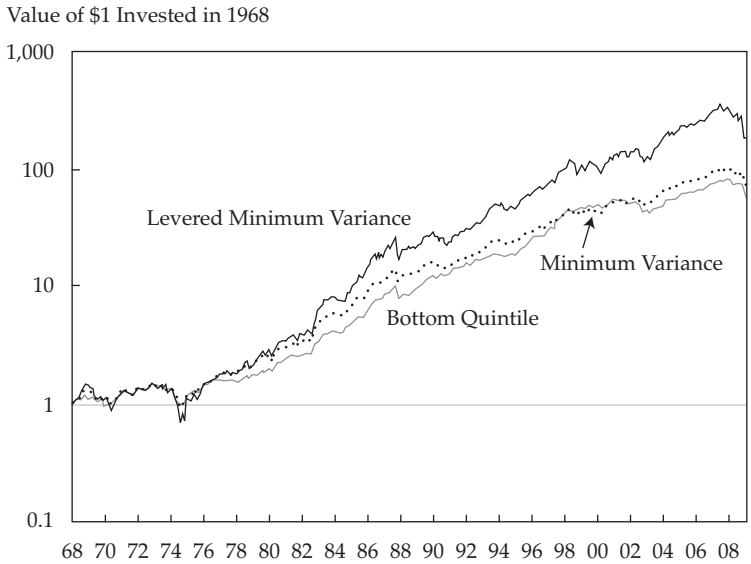
Table 3. A Low-Volatility Portfolio vs. a Portfolio of Low-Volatility Stocks, January 1968–December 2008

	Low-Volatility Quintile	Diagonal Only	Full-Risk Model	Levered, Full-Risk Model
Geometric average $R_p - R_f$	4.12%	5.26%	4.85%	7.26%
Average $R_p - R_f$	4.86%	6.42%	5.41%	8.82%
Standard deviation	12.74%	15.93%	11.50%	18.80%
Sharpe ratio	0.38	0.40	0.47	0.47
Average $R_p - R_m$	0.62%	2.18%	1.17%	4.58%
Tracking error	7.45%	5.61%	8.67%	10.12%
Information ratio	0.08	0.39	0.13	0.45
Beta	0.70	0.95	0.61	1.00
Alpha	2.00%	2.52%	2.90%	4.70%
t(Alpha)	2.03	3.21	3.01	3.03

Notes: For each month, we formed a minimum-variance portfolio of the top 1,000 stocks by market capitalization in the CRSP universe by using two methods and compared performance with a low-volatility sort. We estimated volatility by using up to 60 months of trailing returns (i.e., return data starting as early as January 1963). We estimated the covariance matrix as in Clarke, de Silva, and Thorley (2006). We limited the individual stock weights to between 0 and 3 percent. The third column has a simple five-factor covariance matrix with a Bayesian shrinkage parameter applied to the correlations, and the second column has only the diagonal of the covariance matrix. The fourth column levers the third-column portfolio to produce an average beta of 1. Average returns are monthly averages multiplied by 12. Standard deviation and tracking error are monthly standard deviations multiplied by the square root of 12. The return on the market, R_m, and the risk-free rate, R_f, are from Ken French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

Figure 4. A Low-Volatility Portfolio vs. a Portfolio of Low-Volatility Stocks, January 1968–December 2008



Notes: For each month, we formed portfolios by using the following three methods: (1) A minimum-variance portfolio of the top 1,000 stocks by market capitalization in the CRSP universe under the covariance matrix estimate methodology of Clarke, de Silva, and Thorley (2006). We limited the individual stock weights to between 0 and 3 percent. (2) The portfolio of the lowest quintile by trailing volatility. We measured volatility as the standard deviation of up to 60 months of trailing returns (i.e., return data starting in January 1963). (3) A levered minimum-variance portfolio to produce an average beta of 1.

Conclusion: The Best of Both Worlds

The majority of stock market anomalies can be thought of as "different returns, similar risks." Value and momentum strategies, for example, are of this sort; cross-sectional return differences, not risk differences, are emphasized. Institutional investment managers are well positioned to take advantage of such anomalies because they can generate high excess returns while maintaining average risks, thereby matching their benchmark's risk and controlling tracking error.

But the low-risk anomaly is of a very different character. Exploiting it involves holding stocks with more or less similar long-term returns (which does not help a typical investment manager's excess returns) but with different risks, which only increases tracking error. So, even though irrational investors happily overpay for high risk and shun low risk, investment managers are generally not incentivized to exploit such mispricing. We developed this argument and introduced some preliminary evidence.

Our behavioral finance diagnosis also implies a practical prescription. Investors who want to maximize returns subject to total risk must incentivize their managers to do just that—by focusing on the benchmark-free Sharpe ratio, not the commonly used information ratio. For such investors, our behavioral finance insights are good news because they suggest that, so long as most of the investing world sticks with standard benchmarks, the advantage will be theirs.

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This article qualifies for 1 CE credit.

Appendix A. Delegated Portfolio Management and the CAPM

This short derivation follows Brennan (1993). It shows that delegated portfolio management with a fixed, market benchmark and no leverage will tend to flatten the CAPM relationship, even with no irrational investors, and make low-volatility and low-beta stocks and portfolios components of an attractive investment strategy. We start with assumptions that are sufficient to derive the CAPM.

1. *Stocks and bonds.* There are stocks $i = 1$ to N, with expected returns $\mathbf{R}$ and covariance $\mathbf{\Sigma}$. A risk-free bond returns R_f.
2. *Investors.* There are two representative investors $j = 1, 2$, who are mean–variance utility maximizers over returns with a risk aversion parameter of v.
3. *Investment strategies.* Each representative investor makes a scalar asset allocation decision a_j between stocks and the risk-free asset, as well as a vector portfolio choice decision $\mathbf{w}_j$.
   a. Investor 1 delegates his portfolio choice. He allocates a fraction a_1 of his capital to an intermediary, who chooses a portfolio $\mathbf{w}_1$ on Investor 1's behalf.
   b. Investor 2 chooses her own portfolio. She allocates a fraction a_2 of her capital to stocks and chooses a portfolio $\mathbf{w}_2$, which can be collapsed without loss of generality to a single choice variable $\mathbf{w}_2$. Mean–variance utility maximization means that she chooses $\mathbf{w}_2$ to maximize
   $$E\left(\mathbf{w}_2{}'\mathbf{R} - R_f\mathbf{1}\right) - \tfrac{v}{2}\mathbf{w}_2{}'\mathbf{\Sigma}\mathbf{w}_2.$$
   If there are only Type 2 investors, then the CAPM holds in equilibrium:
   $$E\left(R_i - R_f\right) = \beta_i E\left(R_m - R_f\right). \tag{A1}$$
   If we add Type 1 investors to the model, we need an extra assumption about what intermediaries do. For example, it would be natural to assume that they have an information advantage. To keep the derivation simple, intermediation here simply involves selecting stocks on behalf of Type 1 investors, with the objective of maximizing the portfolio's IR or maximizing returns subject to a tracking error constraint, which is governed by a parameter γ.
4. *Intermediation*. A single intermediary chooses a portfolio $\mathbf{w}_1$ to maximize $E(\mathbf{w}_1 - \mathbf{w}_b)'\mathbf{R} - \gamma(\mathbf{w}_1 - \mathbf{w}_b)'\mathbf{\Sigma}(\mathbf{w}_1 - \mathbf{w}_b)$, where $\mathbf{w}_b$ are the weights in the market portfolio and $(\mathbf{w}_1 - \mathbf{w}_b)'\mathbf{1} = \mathbf{0}$.

Investor 1 allocates a fraction a_1 of his capital to an intermediary. The problem now is that the intermediary no longer cares about maximizing the Sharpe ratio for Type 1 investors. The intermediary chooses $\mathbf{w}_1$ to maximize IR; Investor 2 chooses $\mathbf{w}_2$ to maximize the Sharpe ratio; and the two compete to set prices. Note that the budget constraint in the intermediary's objective means that the IR must be maximized through stock selection (i.e., without resorting to borrowing or

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investing in a risk-free asset). We make no claim that this contract is optimal in the sense of van Binsbergen, Brandt, and Koijen (2008)—only that it is commonly used in practice.

The market must clear so that $a_1\mathbf{w}_1 + a_2\mathbf{w}_2 = \mathbf{w}_b$. Substituting the optimal choices of $\mathbf{w}$ into the market-clearing condition delivers a flattened version of the CAPM:

$$E\left(R_i - R_f\right) = \beta_i E\left(R_m - R_f\right) + c\left(1-\beta_i\right), \qquad \text{(A2)}$$

where

$$c = A\frac{a_1 v}{a_2\gamma + a_1 v} > 0$$

A is a constant that depends on the equilibrium distribution of risk and return and is positive if the Sharpe ratio of the minimum-variance portfolio is positive. Although the amount of capital delegated, a_1, can be easily endogenized and determined as a function of the risk aversion of Type 1 investors, the tracking error mandate (γ), and the investment opportunity set, it does not add much to the intuition of the model. The effects of changes in the other parameters are intuitive. The CAPM relationship is especially flat when γ is small, such that there is a loose tracking error mandate; when Type 1 investors delegate a large amount of capital a_1 to the intermediary; and when Type 2 investors are risk averse or when v is large, leading them to stay out of stocks to a greater extent.

As Brennan (1993) showed, Type 2 investors will specialize in lower-volatility stocks. In this example, they are rational mean–variance utility maximizers who partially offset the effects of an intermediary who tries to capture improvements in the IR by holding higher-volatility stocks. Introducing a set of irrational individual investors with a preference for high volatility will only exacerbate the flattening of the CAPM. Intermediaries will start to act as arbitrageurs only when the relationship between risk and return is inverted.

Notes

1. http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

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Agenda Item #2

INTECH Biographies

Adrian Banner, Ph.D., is chief executive officer and chief investment officer of INTECH. Dr. Banner was named chief executive officer in November 2012 and concurrently is the firm's chief investment officer, a position he has held since January 2012. He joined INTECH in August 2002 and since that time has been an integral part of the firm's Princeton-based research team. In 2009, he was named the firm's co-chief investment officer. Dr. Banner has extensive knowledge of INTECH's trading systems, optimization programs and research initiatives, both on an operational and theoretical basis, and has held various roles as part of INTECH's Princeton research team prior to being named CIO and subsequently CEO. Dr. Banner earned his Ph.D. in mathematics from Princeton University and his M.Sc. and B.Sc., also in mathematics, from the University of New South Wales, Australia. Dr. Banner has lectured on the stability of market capitalization and low-volatility equity portfolios at a number of academic and professional conferences and has authored several scholarly papers on Stochastic Portfolio Theory. His book entitled "The Calculus Lifesaver" has been praised for being "one of the clearest mathematics texts" and "an indispensable volume for any student seeking to master calculus."

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INTECH

The Theory Behind Using Systematic Rebalancing to Capture Natural Stock Price Volatility

Arizona State Retirement System Board
March 26, 2015

Adrian Banner, Ph.D.
Chief Executive Officer and Chief Investment Officer

Warren T. DeKinder, CFA
Managing Director

Christian McCormick, CFA
Managing Director

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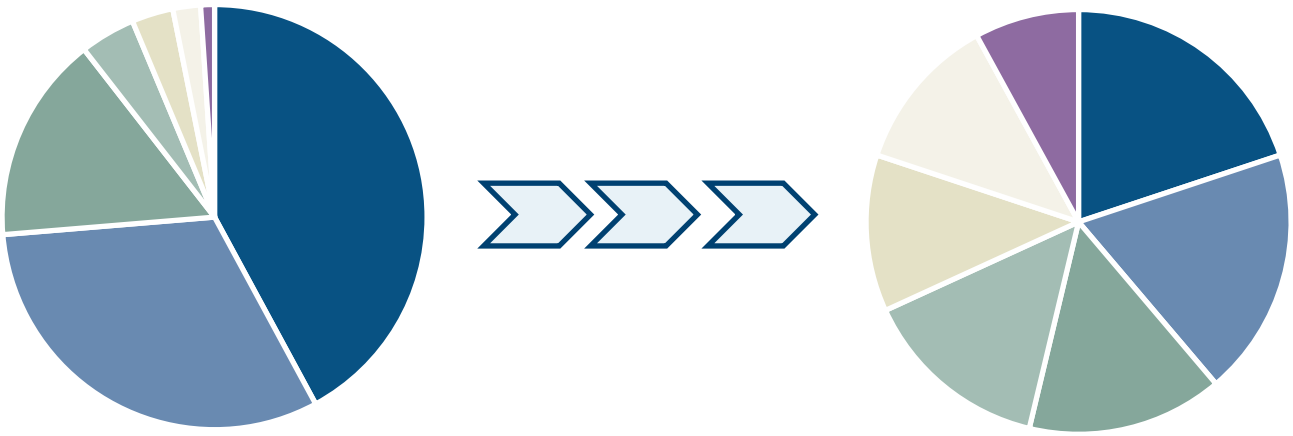
Outline



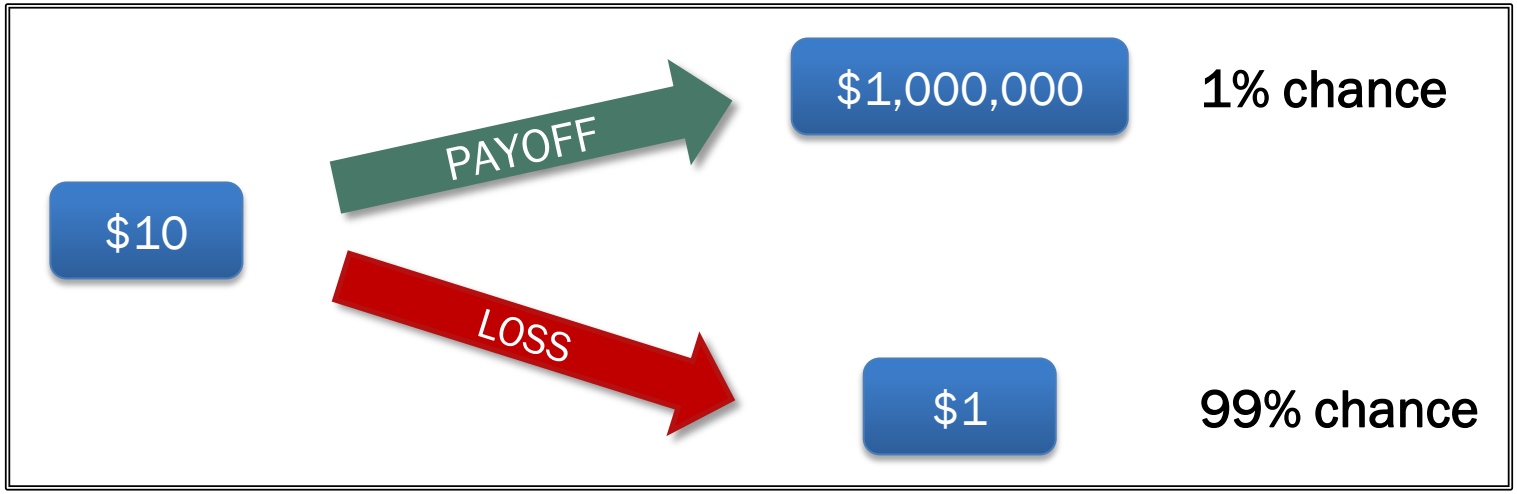
1. **Rebalancing and Diversification**
2. **Illustrative Examples**
3. **Diversity and Market Stability**

Rebalancing and Diversification Improve Compound Return

- "Expected return" means the average return from an investment…

  …but might not actually be what you expect!

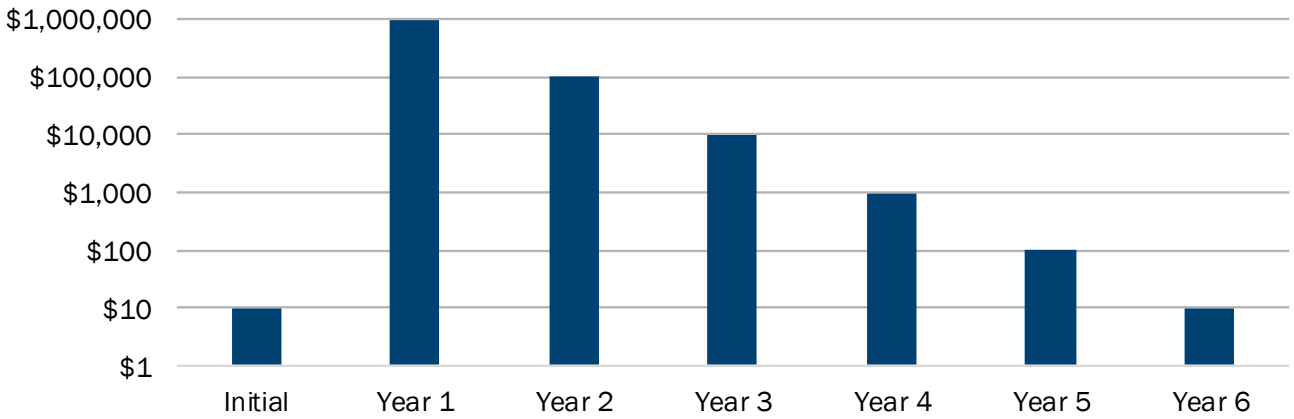
- Hypothetical illustration: a highly speculative investment:



Expected return = 99,909.9%…
but we "expect" -90% most of the time!

Compounding the Example Investment

- Should one invest in the speculative investment?
  - For a single year, it makes sense provided that one can cope with the downside
- What if the investment is compounded over multiple years, reinvesting the proceeds?
  - Even if the investment hits the payoff after year 1, the next five years could reduce the capital back to the original capital:



  - For every five years without a payoff, one needs to hit the payoff just to break even!
  - Since the payoff is only 1/100 (not 1/6), the compound return is poor: -88.5%.

Avoiding the "Curse of Compounding"

- The problem with compounding:

After **positive** returns	⇨	there is more to lose
After **negative** returns	⇨	there is less to work with

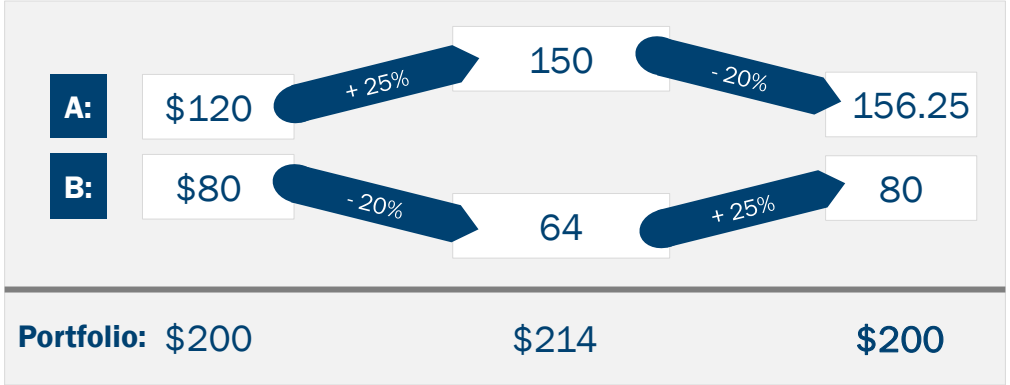
- Diversification helps improve compound return by reducing the above effects.
  - Use 100 uncorrelated versions of the speculative investment…

    …and compound return improves from **-88.5%** to **4,186%**!
  - In this scenario: there is **>63%** chance that at least one of the investments hits the payoff each year.
  - Fail to rebalance…

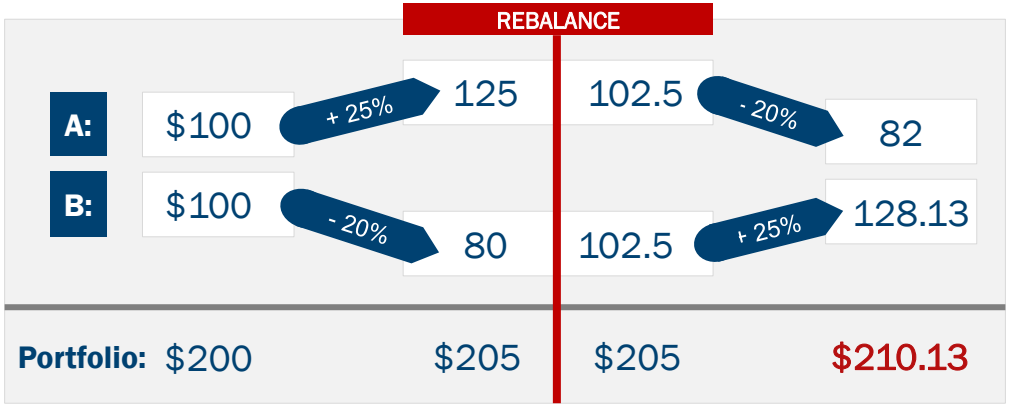
    …and the compound return is **below 0%** again – all diversification is lost.
- Rebalancing to preserve diversification has a buy-low, sell-high character.

Simple Two-Stock Example

Passive Portfolio: No Excess Return
Stock A Return: 0%, Stock B Return: 0%, **Portfolio Return = 0%**



Rebalanced Portfolio: Excess Return
Stock A Return: 0%, Stock B Return: 0%, **Portfolio Return = 5.1%**



Criteria for hypothetical illustration is a two-stock portfolio with perfect negative correlation. Mathematically, a 50%/50% target weighting for a two-stock portfolio with equal growth rates maximizes long-term return. Trading costs and other expenses have not been considered.

A Real-World Example: Equal-Weighted Portfolios

January 1, 1963 – December 31, 2013

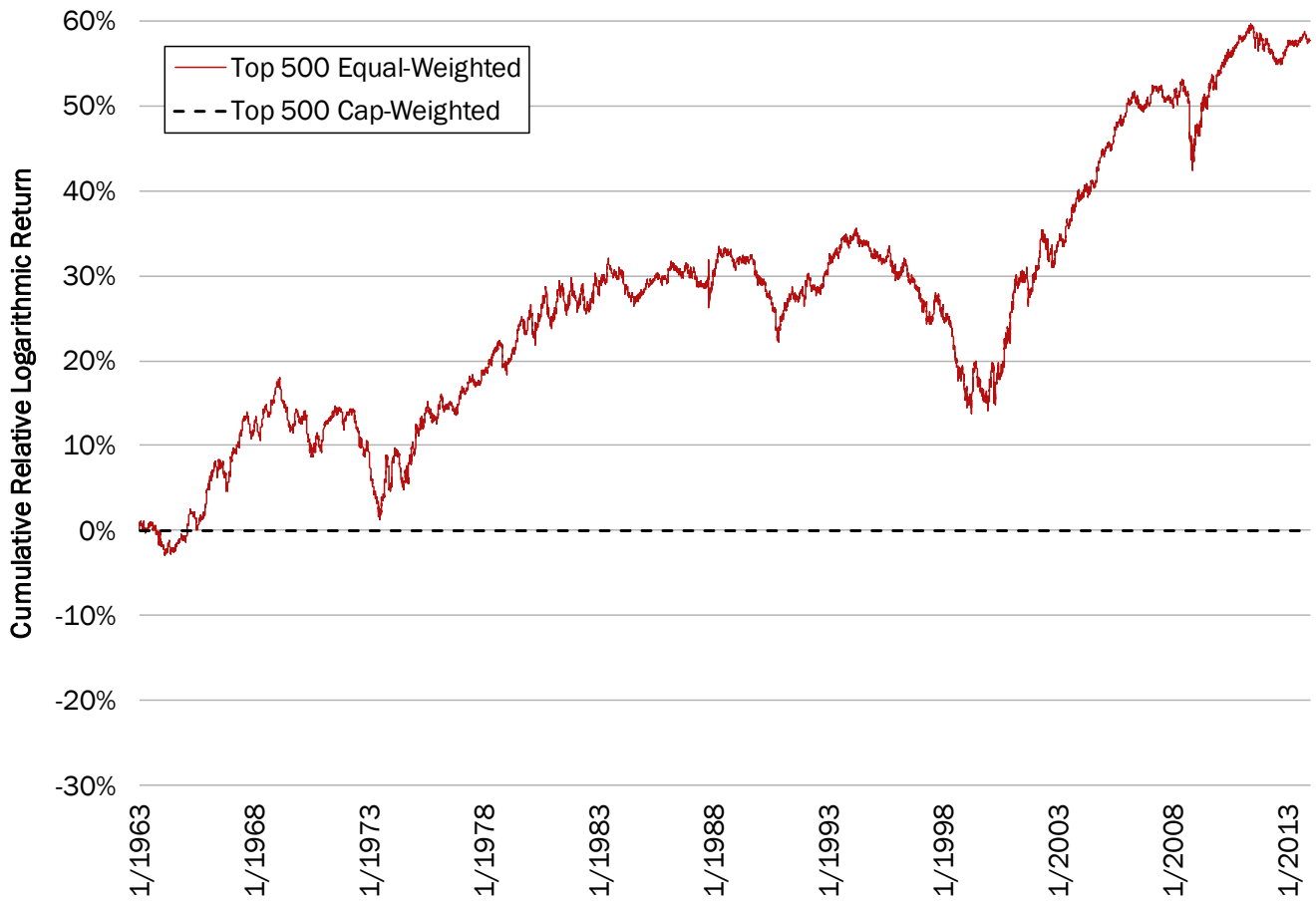


Chart represents the top 500 stocks in the Center for Research in Security Prices (CRSP) U.S. Stock Database based on weighted capitalization for the period shown, rebalanced on a daily basis, assuming no trading costs. Data reflects past performance, which is no guarantee of future results. See Simulations Disclaimer at the end of this presentation for additional information.

A Real-World Example: Equal-Weighted Portfolios (cont'd)

- Equal-weighted portfolios are evidently rebalancing strategies:
  - The S&P 500 Equal Weight Index rebalances each stock to 20 bps quarterly.
  - Apart from corporate actions, these trades are *all* buy-low, sell-high.
  - Is this enough to explain the historical outperformance?
- Equal-weighted portfolios are also systematically overweighted in smaller-cap stocks.
  - This is widely believed to be the source of outperformance ("size risk premium").
  - There is little direct evidence for this effect, however!

The Source of Excess Return in an Equal-Weighted Portfolio

Three tests consistent with the hypothesis that rebalancing is the source of the excess return:

1. Stock compound return conditioned on capitalization "rank" (e.g. Apple is rank 1, Exxon is rank 2, ...) shows no systematic bias in favor of smaller-cap names.

2. Less frequent rebalancing causes a drop in excess return without materially reducing size exposure.

3. The rebalancing premium can be computed directly in this case using Stochastic Portfolio Theory.

Stock Relative Growth Rate by Rank

Average growth rank based on capitalization rank, U.S. Equity Market
January 2, 1964 – December 31, 2012

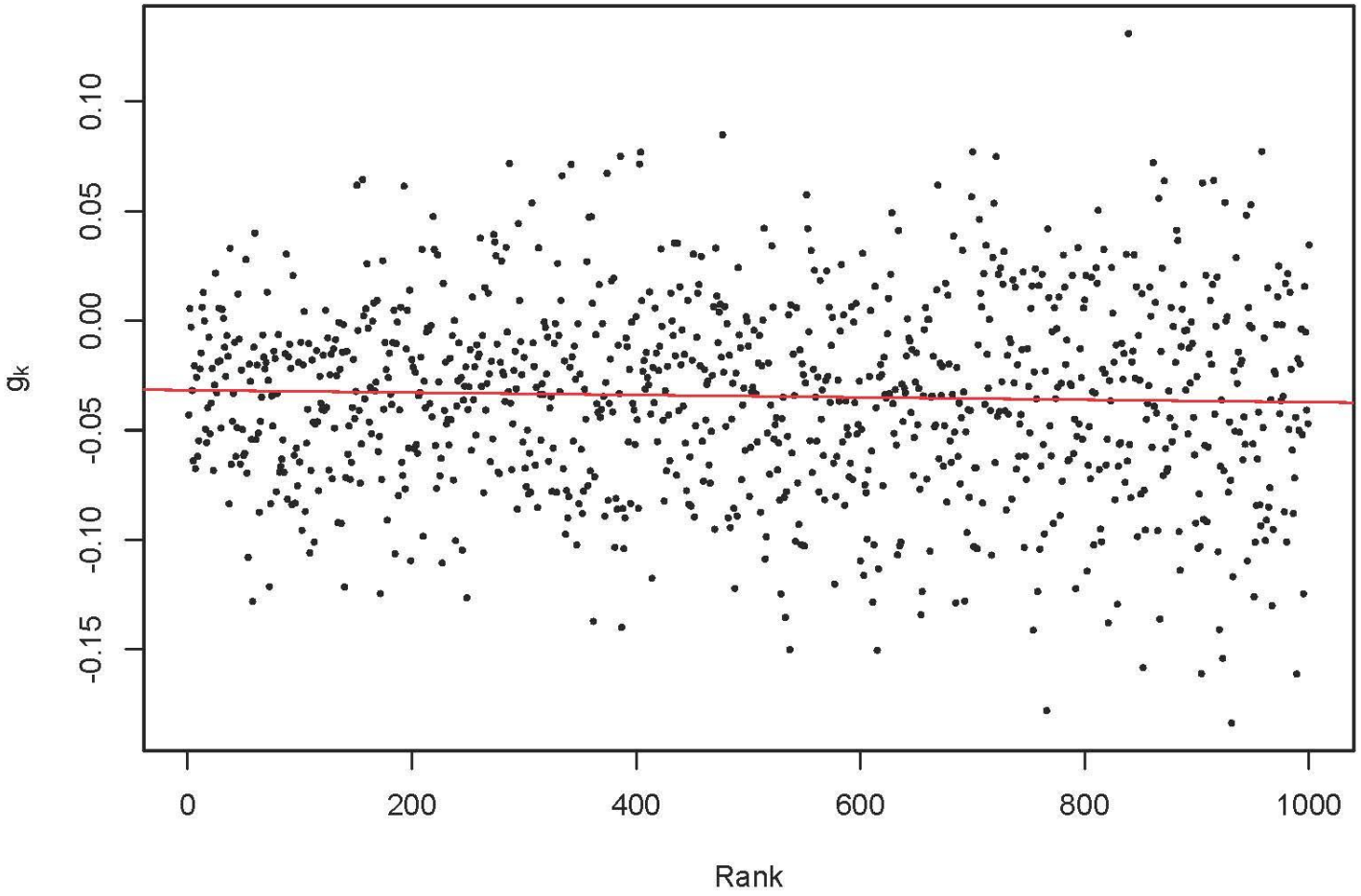
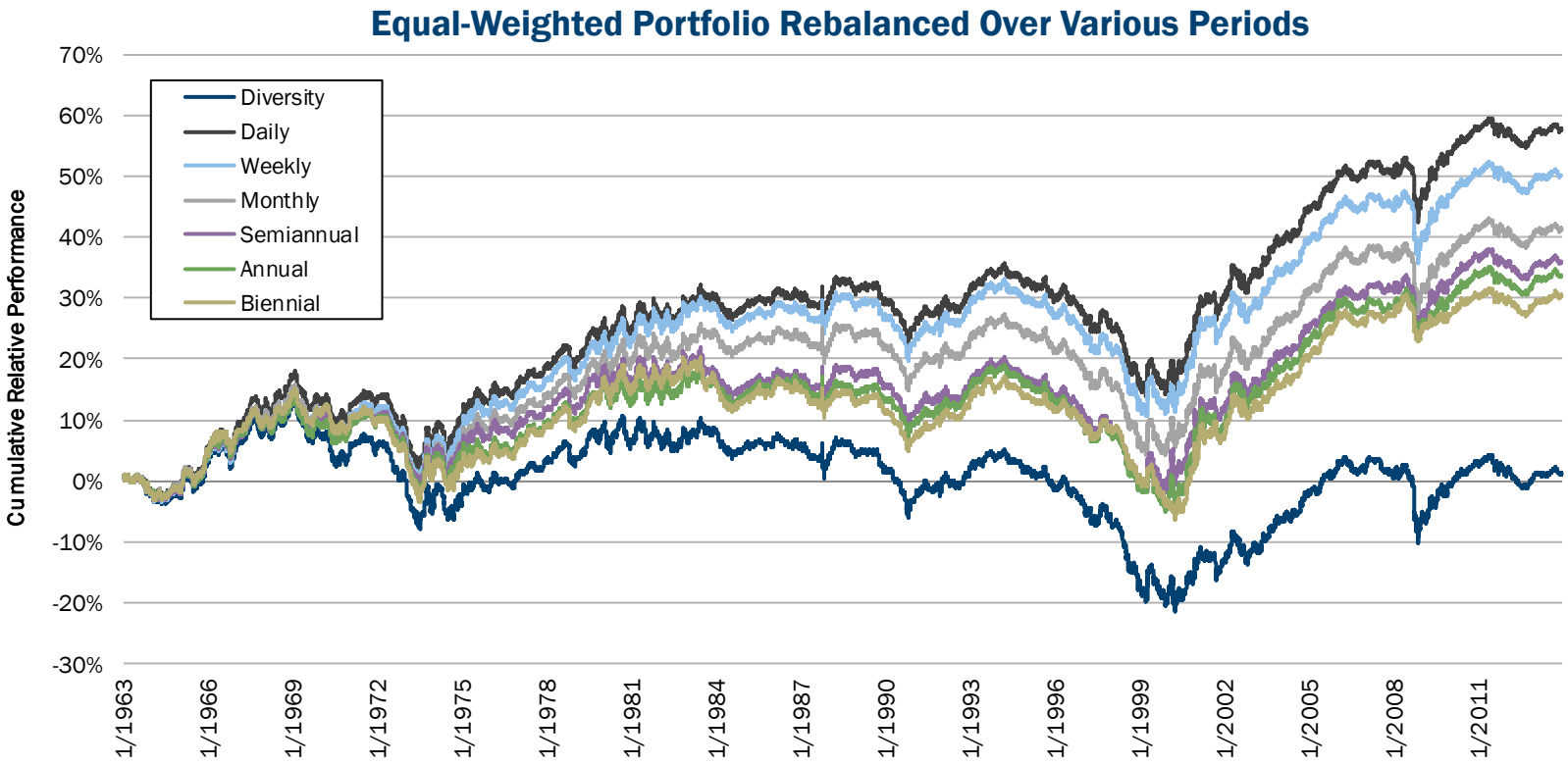


Chart represents the cumulative logarithmic relative return of the top 100 U.S. stocks by market capitalization versus the top 1,000 U.S. stocks by market capitalization for the period January 1, 1964 to December 31, 2012 rebalanced on a monthly basis, assuming no trading costs. Data source: Center for Research in Security Prices (CRSP) U.S. Stock Database. Data reflects past performance, which is no guarantee of future results.

Effect of Rebalancing on an Equal-Weighted Portfolio

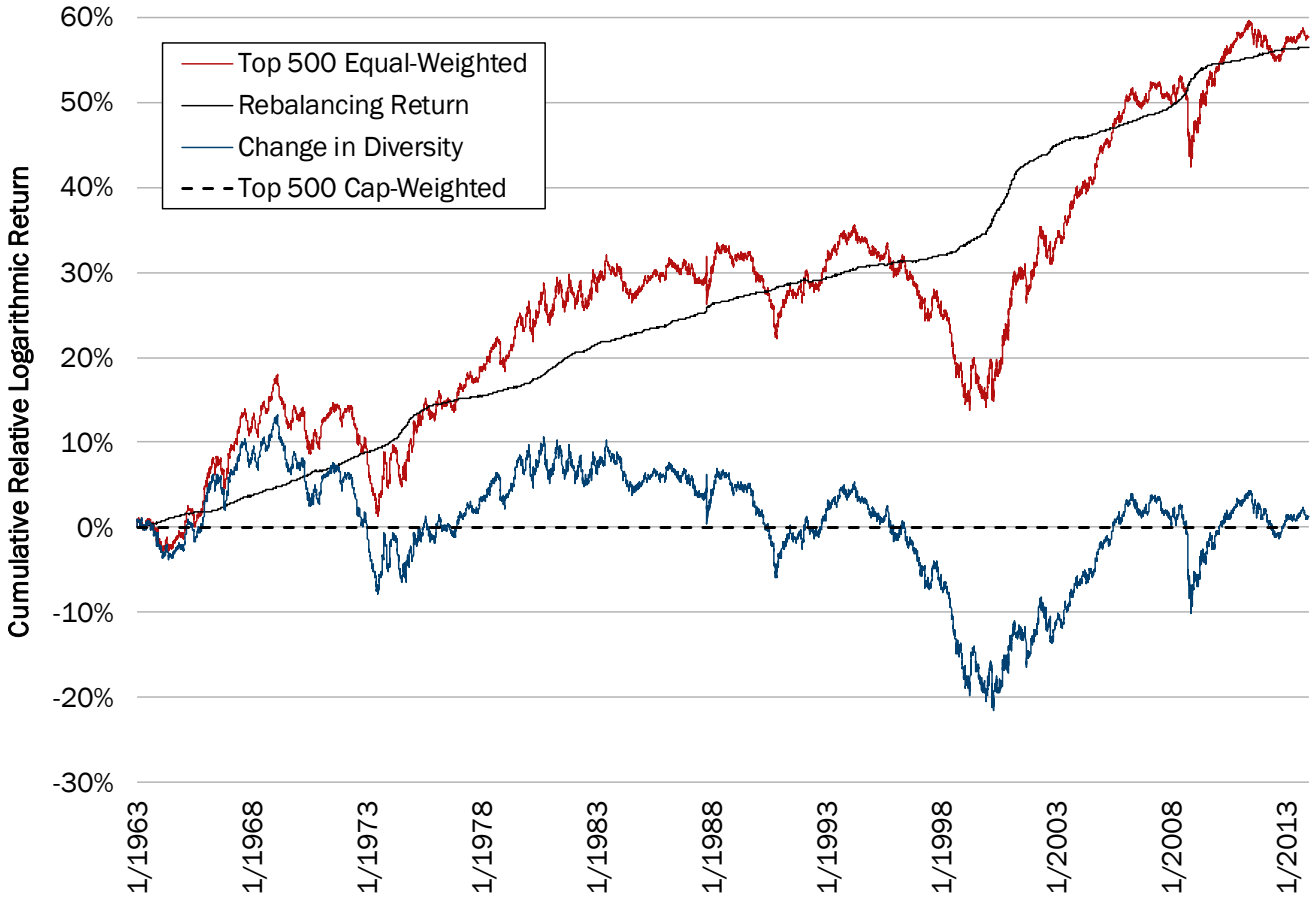
January 1, 1963 - December 31, 2013



Calculation utilizes CRSP data. The market is reconstituted every period (daily, weekly, etc.) from the top 500 stocks in the universe at the time. The market and the equal-weighted portfolio are buy-and-hold between reconstitutions, but the performance and the effect of diversity is computed daily. In particular, all returns are simulated. Results include the reinvestment of all dividends, interest, and capital gains, but do not reflect deduction of investment advisory fees or trading costs. Thus, returns will be reduced by these and any other expenses. Past performances cannot guarantee future results. Although the information contained herein has been obtained from sources believed to be reliable, its accuracy and completeness cannot be guaranteed. See Simulations Disclaimer at the end of this presentation for additional information.

Computing the Rebalancing Premium

January 1, 1963 – December 31, 2013



The return due to rebalancing (black line) equals the relative return (red line) minus the change in diversity (blue line).

Chart represents the top 500 stocks in the Center for Research in Security Prices (CRSP) U.S. Stock Database based on weighted capitalization for the period shown, rebalanced on a daily basis, assuming no trading costs. Data reflects past performance, which is no guarantee of future results. See Simulations Disclaimer at the end of this presentation for additional information.

Diversity and Market Stability

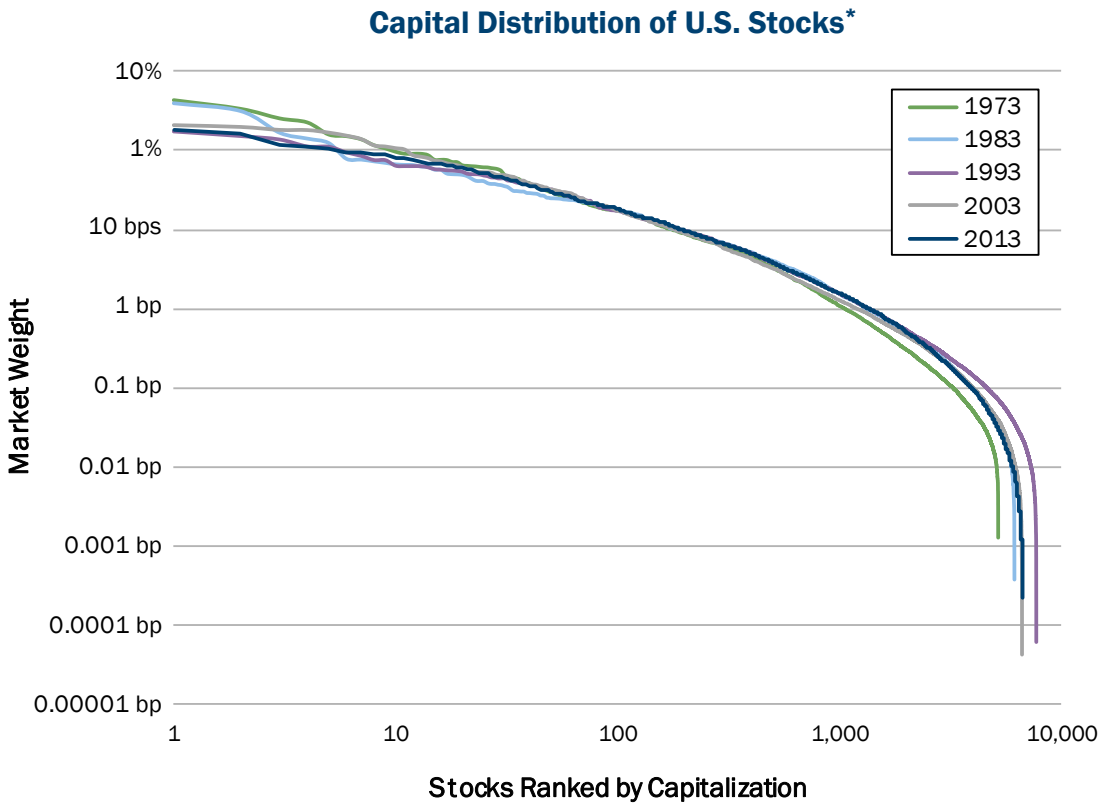
- No individual stock mean-reversion is required, and little is likely to be reliably present for S&P 500 Index names.
- Instead, market stability is important. Diversity measures stability:

  Diversity ↑ = **smaller** stocks in the S&P 500 increase their market share.

  Diversity ↓ = **larger** stocks in the S&P 500 increase their market share.

- If diversity continues to decrease, a few stocks drive most of the index return.
  - This is a poor environment for rebalancing strategies to outperform the market.
  - However, diversity has historically been mean-reverting.

Distribution of Capital

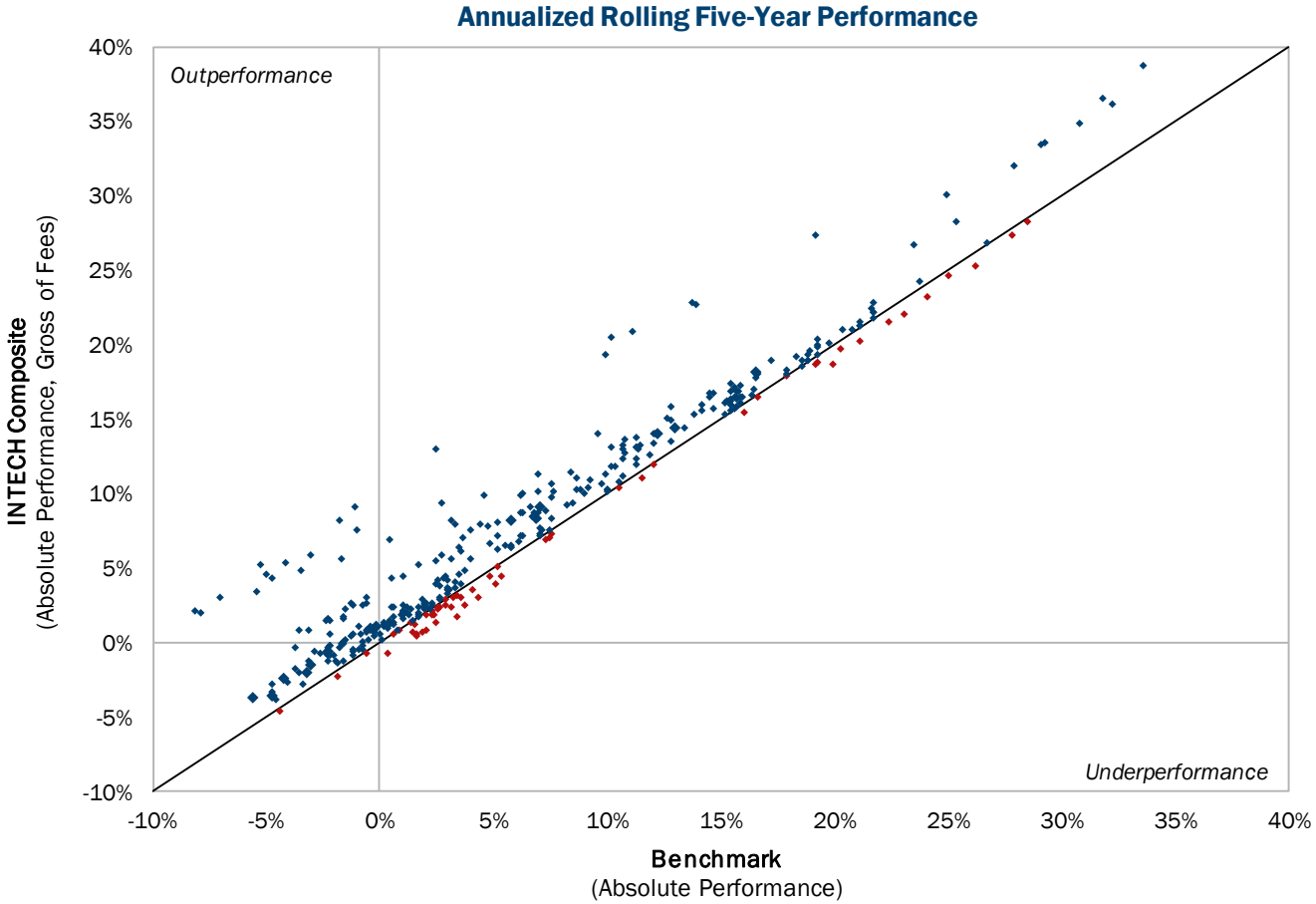


Size effects tend to average out over time.

*The curves were generated using the capitalization data from the daily stock database of the Center for Research in Securities Prices (CRSP) U.S. Stock Database. The market at each snapshot consists of the constituents of the CRSP universe with available capitalization data on the last business day of the years shown. The market weight of a stock is defined to be the ratio of its market capitalization to the total market capitalization of all stocks in the market. Stocks are log-ranked by capitalization from the largest stock (rank 1) to the smallest stock (rank <10,000).

Rebalancing in Practice

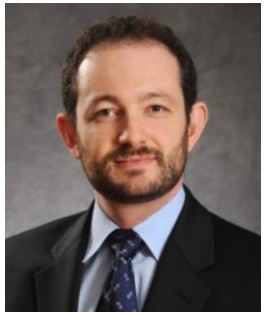
INTECH Relative-Risk Strategy Composites vs. Benchmark
July 1, 1987 – December 31, 2014



The rebalancing strategies outperformed 323 of 379 periods, or 85% of the time (12 composites, gross of fees).

Older rolling periods reflect fewer strategies than more recent rolling periods. Performance for rolling periods other than five years are different and are available upon request. Data reflects past performance, which does not guarantee future results. Performance includes the reinvestment of dividends and other earnings. Outperformance is not indicative of positive absolute performance. Rolling periods calculated quarterly since each composite's inception. Chart includes performance for all active relative-risk strategies with at least a three-year track record, and excludes terminated published composites. See Composite Performance for standardized performance and Presentation Notes for additional information on the strategies, inception dates, and benchmarks included in this chart.

Presenter Bios



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Presenter Bios (cont'd)

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APPENDIX

INTECH Strategies: Composite Performance

As of December 31, 2014

Active Risk-Managed U.S. Strategies	QTD	1 Yr	3 Yrs	5 Yrs	7 Yrs	10 Yrs	15 Yrs	20 Yrs	ITD	Inception Date
			Annualized Returns							
U.S. Enhanced Plus Gross	6.36%	15.43%	20.87%	16.34%	7.83%	8.60%	6.22%	11.34%	11.06%	7/1/1987
S&P 500 Index	4.93%	13.69%	20.41%	15.45%	7.27%	7.67%	4.24%	9.85%	9.65%	
Difference (Gross-Index)	1.43%	1.74%	0.46%	0.88%	0.56%	0.92%	1.97%	1.48%	1.41%	
U.S. Enhanced Plus Net	6.29%	15.10%	20.52%	15.99%	7.51%	8.27%	5.86%	10.95%	10.67%	
U.S. Large Cap Growth Gross	3.11%	10.32%	19.48%	15.43%	8.15%	8.14%	6.38%	13.70%	12.86%	7/1/1993
S&P 500 Growth Index*	5.06%	14.89%	20.46%	16.05%	8.78%	8.48%	3.02%	9.95%	9.60%	
Difference (Gross-Index)	-1.95%	-4.57%	-0.98%	-0.62%	-0.63%	-0.34%	3.36%	3.75%	3.26%	
U.S. Large Cap Growth Net	2.99%	9.80%	18.93%	14.89%	7.65%	7.64%	5.86%	13.13%	12.28%	
U.S. Enhanced Index Gross	4.78%	13.00%	19.83%	15.54%	7.67%	8.17%	5.24%		6.71%	4/1/1998
S&P 500 Index	4.93%	13.69%	20.41%	15.45%	7.27%	7.67%	4.24%		5.74%	
Difference (Gross-Index)	-0.16%	-0.69%	-0.58%	0.09%	0.40%	0.49%	1.00%		0.97%	
U.S. Enhanced Index Net	4.70%	12.66%	19.46%	15.18%	7.33%	7.83%	4.90%		6.37%	
U.S. Broad Large Cap Growth Gross	7.45%	13.12%	21.29%	16.93%	7.93%	8.35%			4.98%	11/1/2000
Russell 1000 Growth Index	4.78%	13.05%	20.26%	15.81%	8.41%	8.49%			2.79%	
Difference (Gross-Index)	2.67%	0.07%	1.03%	1.12%	-0.48%	-0.14%			2.19%	
U.S. Broad Large Cap Growth Net	7.32%	12.57%	20.69%	16.35%	7.39%	7.81%			4.44%	
U.S. Broad Enhanced Plus Gross	6.07%	13.08%	20.57%	16.43%	7.78%	8.34%			7.92%	4/1/2001
Russell 1000 Index	4.88%	13.24%	20.62%	15.64%	7.48%	7.96%			6.71%	
Difference (Gross-Index)	1.19%	-0.16%	-0.05%	0.79%	0.30%	0.38%			1.22%	
U.S. Broad Enhanced Plus Net	5.99%	12.74%	20.20%	16.07%	7.45%	8.02%			7.58%	
U.S. Large Cap Core Gross	7.16%	14.72%	20.56%	16.02%	7.62%	8.47%			7.85%	8/1/2001
S&P 500 Index	4.93%	13.69%	20.41%	15.45%	7.27%	7.67%			6.13%	
Difference (Gross-Index)	2.23%	1.03%	0.14%	0.57%	0.35%	0.80%			1.73%	
U.S. Large Cap Core Net	7.03%	14.15%	19.98%	15.46%	7.10%	7.96%			7.35%	
U.S. Broad Large Cap Value Gross	4.43%	9.96%	20.51%	16.21%	7.42%	8.14%			9.26%	8/1/2004
Russell 1000 Value Index	4.98%	13.45%	20.89%	15.42%	6.45%	7.30%			8.32%	
Difference (Gross-Index)	-0.55%	-3.49%	-0.39%	0.79%	0.97%	0.84%			0.94%	
U.S. Broad Large Cap Value Net	4.34%	9.56%	20.06%	15.78%	7.02%	7.74%			8.85%	
U.S. Broad Enhanced Index Gross	4.63%	11.66%	20.15%	15.68%					12.03%	10/1/2008
Russell 1000 Index	4.88%	13.24%	20.62%	15.64%					12.25%	
Difference (Gross-Index)	-0.25%	-1.58%	-0.47%	0.04%					-0.22%	
U.S. Broad Enhanced Index Net	4.57%	11.42%	19.89%	15.41%					11.74%	
Enhanced Index North America Gross	3.56%	11.32%							19.34%	7/1/2012
MSCI North America Index	4.15%	12.57%							19.60%	
Difference (Gross-Index)	-0.59%	-1.25%							-0.27%	
Enhanced Index North America Net	3.49%	11.01%							19.00%	

*S&P 500 Growth Index represents the S&P 500/Barra methodology from inception to 2005, S&P 500/Citigroup Growth Index methodology from 2006 through 12/8/09 and S&P 500 Growth Index methodology thereafter.
Past performance does not guarantee future results. Performance includes the reinvestment of dividends and other earnings. Difference may not agree with input data due to rounding. See Presentation Notes for additional information.

INTECH Strategies: Composite Performance (cont'd)

As of December 31, 2014

Active Risk-Managed U.S. Strategies	QTD	1 Yr	Annualized Returns 3 Yrs	5 Yrs	7 Yrs	10 Yrs	15 Yrs	20 Yrs	ITD	Inception Date
Large Cap Core USA Gross	3.17%	11.05%							20.52%	8/1/2012
MSCI USA Index	4.81%	13.36%							20.66%	
Difference (Gross-Index)	-1.64%	-2.31%							-0.14%	
Large Cap Core USA Net	3.06%	10.58%							20.03%	
Global/Non-U.S. Strategies										
Global Large Cap Core Gross	1.82%	5.30%	17.75%	12.67%	5.30%	8.36%			8.36%	1/1/2005
MSCI World Index	1.12%	5.50%	16.13%	10.81%	3.87%	6.61%			6.61%	
Difference (Gross-Index)	0.70%	-0.21%	1.62%	1.86%	1.44%	1.76%			1.76%	
Global Large Cap Core Net	1.71%	4.83%	17.21%	12.14%	4.78%	7.81%			7.81%	
International Large Cap Core Gross	-2.16%	-4.10%	14.06%	8.17%	1.63%				4.55%	11/1/2006
MSCI EAFE Index	-3.53%	-4.49%	11.56%	5.81%	0.00%				2.12%	
Difference (Gross-Index)	1.37%	0.39%	2.51%	2.36%	1.63%				2.43%	
International Large Cap Core Net	-2.26%	-4.50%	13.59%	7.71%	1.21%				4.11%	
Global Large Cap Core ex Japan (Kokusai) Gross	1.58%	6.20%	18.51%	13.16%					16.94%	5/1/2009
MSCI Kokusai (World ex Japan) Index	1.44%	6.38%	16.72%	11.33%					15.95%	
Difference (Gross-Index)	0.14%	-0.18%	1.78%	1.83%					0.99%	
Global Large Cap Core ex Japan (Kokusai) Net	1.47%	5.75%	17.92%	12.57%					16.34%	
European Large Cap Core (EUR) Gross	1.71%	8.51%	19.36%	14.04%					14.04%	1/1/2010
MSCI Europe Index (EUR)	-0.10%	7.40%	15.19%	9.58%					9.58%	
Difference (Gross-Index)	1.80%	1.11%	4.17%	4.46%					4.46%	
European Large Cap Core (EUR) Net	1.58%	7.94%	18.71%	13.42%					13.42%	
Global All Country Enhanced Index Gross	0.60%	4.35%	15.15%						13.50%	11/1/2011
MSCI All Country World Index	0.52%	4.70%	14.72%						12.76%	
Difference (Gross-Index)	0.08%	-0.35%	0.43%						0.74%	
Global All Country Enhanced Index Net	0.51%	3.99%	14.75%						13.11%	
Global High Dividend Core Gross	-0.71%	1.88%							11.84%	4/1/2012
MSCI World High Dividend Yield Index	-1.52%	3.28%							11.98%	
Difference (Gross-Index)	0.81%	-1.40%							-0.14%	
Global High Dividend Core Net	-0.84%	1.32%							11.23%	
Global Enhanced Index Gross	0.95%	5.21%							19.03%	6/1/2012
MSCI World Index	1.12%	5.50%							18.46%	
Difference (Gross-Index)	-0.17%	-0.29%							0.57%	
Global Enhanced Index Net	0.87%	4.85%							18.63%	

Past performance does not guarantee future results. Performance includes the reinvestment of dividends and other earnings. Difference may not agree with input data due to rounding. See Presentation Notes for additional information.

INTECH Strategies: Composite Performance (cont'd)

As of December 31, 2014

Active Risk-Managed Global/Non-U.S. Strategies	QTD	1 Yr	Annualized Returns 3 Yrs	5 Yrs	7 Yrs	10 Yrs	15 Yrs	20 Yrs	ITD	Inception Date
Global All Country Core Gross	0.96%	4.20%							10.53%	5/1/2013
MSCI All Country World Index	0.52%	4.70%							10.31%	
Difference (Gross-Index)	**0.44%**	**-0.50%**							**0.22%**	
Global All Country Core Net	0.84%	3.64%							9.90%	
Global All Country Core Select Gross	1.32%	4.73%							11.61%	6/1/2013
MSCI All Country World Index	0.52%	4.70%							11.01%	
Difference (Gross-Index)	**0.80%**	**0.02%**							**0.60%**	
Global All Country Core Select Net	1.15%	4.02%							10.85%	
Emerging Markets Core Gross	-2.62%	-1.63%							-1.51%	6/1/2013
MSCI Emerging Markets Index	-4.44%	-1.82%							-0.49%	
Difference (Gross-Index)	**1.82%**	**0.19%**							**-1.02%**	
Emerging Markets Core Net	-2.81%	-2.41%							-2.29%	
Global Defensive Core Gross	4.36%								6.54%	4/1/2014
Russell Global Large Cap Defensive Index	1.04%								4.38%	
Difference (Gross-Index)	**3.32%**								**2.16%**	
Global Defensive Core Net	4.22%								6.10%	
Global All Country Core ex U.S. Gross	-2.51%								-8.20%	7/1/2014
MSCI All Country World ex USA Index	-3.81%								-8.81%	
Difference (Gross-Index)	**1.31%**								**0.61%**	
Global All Country Core ex U.S. Net	-2.65%								-8.48%	
Absolute-Risk Strategies*										
Global Low Volatility Gross	5.65%	11.20%	15.44%						15.44%	1/1/2012
MSCI World Index	1.12%	5.50%	16.13%						16.13%	
Difference (Gross-Index)	**4.53%**	**5.70%**	**-0.69%**						**-0.69%**	
Global Low Volatility Net	5.55%	10.81%	15.03%						15.03%	
U.S. Defensive Managed Volatility Gross	8.81%	15.93%							19.35%	7/1/2012
Russell 1000 Defensive Index	5.39%	13.80%							18.92%	
Difference (Gross-Index)	**3.42%**	**2.14%**							**0.43%**	
U.S. Defensive Managed Volatility Net	8.66%	15.32%							18.71%	
U.S. Low Volatility Gross	8.31%	17.95%							18.49%	8/1/2012
Russell 1000 Index	4.88%	13.24%							21.01%	
Difference (Gross-Index)	**3.43%**	**4.71%**							**-2.52%**	
U.S. Low Volatility Net	8.23%	17.57%							18.10%	

*INTECH's absolute-risk strategies are engineered to reduce the portfolio's absolute standard deviation for a given target excess return rather than its tracking error relative to a cap-weighted benchmark.
Past performance does not guarantee future results. Performance includes the reinvestment of dividends and other earnings. Difference may not agree with input data due to rounding. See Presentation Notes for additional information.

INTECH Strategies: Composite Performance (cont'd)

As of December 31, 2014

Active Risk-Managed Absolute-Risk Strategies*	QTD	1 Yr	Annualized Returns 3 Yrs	5 Yrs	7 Yrs	10 Yrs	15 Yrs	20 Yrs	ITD	Inception Date
Eurozone Low Volatility (EUR) Gross	3.54%	9.97%							13.47%	8/1/2012
MSCI EMU Index (EUR)	-0.11%	5.08%							17.47%	
Difference (Gross-Index)	**3.66%**	**4.89%**							**-4.00%**	
Eurozone Low Volatility (EUR) Net	3.45%	9.58%							13.08%	
Emerging Markets Managed Volatility Gross	-5.78%	-4.33%							-1.80%	4/1/2013
MSCI Emerging Markets Index	-4.44%	-1.82%							-1.44%	
Difference (Gross-Index)	**-1.34%**	**-2.51%**							**-0.35%**	
Emerging Markets Managed Volatility Net	-5.97%	-5.09%							-2.58%	
Emerging Markets Low Volatility Gross	-4.66%	-1.58%							-1.10%	6/1/2013
MSCI Emerging Markets Index	-4.44%	-1.82%							-0.49%	
Difference (Gross-Index)	**-0.22%**	**0.24%**							**-0.60%**	
Emerging Markets Low Volatility Net	-4.79%	-2.11%							-1.64%	
U.S. Managed Volatility Gross	7.21%	15.81%							19.80%	6/1/2013
Russell 1000 Index	4.88%	13.24%							18.33%	
Difference (Gross-Index)	**2.33%**	**2.57%**							**1.48%**	
U.S. Managed Volatility Net	7.06%	15.17%							19.15%	
Global All Country Managed Volatility ex Australia (AUD) Gross	10.57%	18.38%							26.07%	10/1/2013
MSCI All Country World ex Australia Index (AUD)	7.60%	14.71%							22.66%	
Difference (Gross-Index)	**2.96%**	**3.68%**							**3.41%**	
Global All Country Managed Volatility ex Australia (AUD) Net	10.40%	17.68%							25.32%	
Global Defensive Managed Volatility Gross	5.68%								10.01%	4/1/2014
Russell Global Large Cap Defensive Index	1.04%								4.38%	
Difference (Gross-Index)	**4.65%**								**5.63%**	
Global Defensive Managed Volatility Net	5.53%								9.52%	
Global Managed Volatility Gross	4.20%								2.93%	7/1/2014
MSCI World Index	1.12%								-0.95%	
Difference (Gross-Index)	**3.08%**								**3.88%**	
Global Managed Volatility Net	4.06%								2.65%	
Global Managed Volatility (EUR) Gross	7.86%								14.11%	7/1/2014
MSCI World Index (EUR)	5.57%								12.07%	
Difference (Gross-Index)	**2.30%**								**2.04%**	
Global Managed Volatility (EUR) Net	7.72%								13.79%	
Global All Country Managed Volatility Gross	3.91%								3.91%	10/1/2014
MSCI All Country World Index	0.52%								0.52%	
Difference (Gross-Index)	**3.39%**								**3.39%**	
Global All Country Managed Volatility Net	3.76%								3.76%	

*INTECH's absolute-risk strategies are engineered to reduce the portfolio's absolute standard deviation for a given target excess return rather than its tracking error relative to a cap-weighted benchmark.
Past performance does not guarantee future results. Performance includes the reinvestment of dividends and other earnings. Difference may not agree with input data due to rounding. See Presentation Notes for additional information.

Presentation Notes

INTECH, an indirect subsidiary of Janus Capital Group Inc., is an investment adviser registered under the Investment Advisers Act of 1940 utilizing a mathematically-based, risk managed investment process that attempts to capitalize on volatility in stock price movements. INTECH is affiliated with Janus Capital Group Inc. and its subsidiaries and affiliates. These subsidiaries and/or affiliates include Janus Capital Management LLC and Perkins Investment Management **LLC. Past performance cannot guarantee future results. Investing involves risk, including the possible loss of principal and fluctuation of value.** In addition, the proprietary mathematical investment process used by INTECH may not achieve the desired results. Performance results reflect the reinvestment of dividends and other earnings. Portfolio performance results shown are time-weighted rates of return using daily valuation, include the effect of transaction costs (commissions, exchange fees, etc.), and are gross of non-reclaimable withholding taxes, if any. The composite includes all actual fee-paying accounts managed on a fully discretionary basis according to the investment strategy from inception date, including those no longer under management. Accounts meeting such criteria enter the composite upon the full first month under management. For periods of less than one year, performance is not annualized. Reporting currency is USD unless otherwise noted. INTECH claims compliance with the Global Investment Performance Standards (GIPS®). To receive INTECH's complete list of composite descriptions and/or presentations that adhere to the GIPS standards, please contact INTECH at Finance@intechjanus.com.

The gross performance results presented do not reflect the deduction of investment advisory fees. Returns will be reduced by such advisory fees and other contractual expenses as described in each client's individual contract.

The net performance results do not reflect the deduction of investment advisory fees actually charged to the accounts in the composite. However, the net performance results do reflect the deduction of model investment advisory fees. Through December 31, 2004, net returns were derived using the maximum fixed fee in effect for each strategy. As of January 1, 2005, net returns are calculated by applying the standard fee schedule in effect for the respective period to each account in the composite on a monthly basis. Actual advisory fees may vary among clients invested in this strategy. Some clients may utilize a performance-based fee. Actual advisory fees paid may be higher or lower than model advisory fees.

The U.S. Enhanced Plus strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the S&P 500 Index. The long-term target excess return objective over the benchmark is 175 – 200 basis points (annualized gross of fees). The composite was created in January 1993.

The U.S. Large Cap Growth strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization growth securities. The benchmark is the S&P 500 Growth Index. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The composite was created in July 1993.

The U.S. Enhanced Index strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the S&P 500 Index. The long-term target excess return objective is 125 – 135 basis points (annualized gross of fees). The composite was created in April 1998.

The U.S. Broad Large Cap Growth strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization growth securities. The benchmark is the Russell 1000 Growth Index. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The composite was created in November 2000.

The U.S. Broad Enhanced Plus strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the Russell 1000 Index. The long-term target excess return objective is 175 – 200 basis points (annualized gross of fees). The composite was created in April 2001.

The U.S. Large Cap Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the S&P 500 Index. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). From January 1997 through June 2001, the composite contained one non-fee paying account managed for the then parent company. The composite was created in January 1993.

The U.S. Broad Large Cap Value strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization value securities. The benchmark is the Russell 1000 Value Index. The long-term target excess return objective is 175 – 225 basis points (annualized gross of fees). The composite was created in August 2004.

The U.S. Broad Enhanced Index strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the Russell 1000 Index. The long-term target excess return objective is 125 – 135 basis points (annualized gross of fees). The composite was created in March 2006.

The Enhanced Index North America strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the MSCI North America Index. The long-term target excess return objective is 115 - 125 basis points (annualized gross of fees). The composite was created in July 2012.

The Large Cap Core USA strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the MSCI USA Index. The long-term target excess return objective over the benchmark is 250 - 300 basis points (annualized gross of fees). The composite was created in August 2012.

The Global Large Cap Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI World Index. The long-term target excess return objective is 250 – 350 basis points (annualized gross of fees). The composite was created in January 2005.

The International Large Cap Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI EAFE Index. The long-term target excess return objective is 300 – 350 basis points (annualized gross of fees). The composite was created in November 2006.

The Global Large Cap Core ex Japan (Kokusai) strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI World ex Japan Index. The long-term target excess return objective is 250 – 350 basis points (annualized gross of fees). The composite was created in May 2009.

The European Large Cap Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI Europe Index. The long-term target excess return objective is 300 – 350 basis points (annualized gross of fees). The composite was created in January 2010.

The Global All Country Enhanced Index strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI All Country World Index. The long-term target excess return objective over the benchmark is 125 – 150 basis points (annualized gross of fees). The composite was created in November 2011.

The Global High Dividend Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI World High Dividend Yield Index. The long-term target excess return objective is 250 – 300 basis points (annualized gross of fees). The composite was created in April 2012.

The Global Enhanced Index strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI World Index. The long-term target excess return objective is 125 – 150 basis points (annualized gross of fees). The composite was created in June 2012.

The Global All Country Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI All Country World Index. The long-term target excess return objective is 250 – 350 basis points (annualized gross of fees). The composite was created in June 2013.

The Global All Country Core Select strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI All Country World Index. The long-term target excess return objective is 375 – 425 basis points (annualized gross of fees). The composite was created in June 2013.

The Emerging Markets Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI Emerging Markets Index. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The composite was created in June 2013.

The Global Defensive Core strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the Russell Global Large Cap Defensive Index. The long-term target excess return objective is 250 - 350 basis points (annualized gross of fees). The composite was created in May 2014.

The Global All Country Core ex U.S. strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI All Country World ex USA Index. The long-term target excess return objective is 250 – 350 basis points (annualized gross of fees). The composite was created in July 2014.

The Global Low Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI World Index. The strategy targets modest above-market returns, with significantly lower standard deviation than the index and a higher Sharpe Ratio, over the long-term. The composite was created in January 2012.

The U.S. Defensive Managed Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the Russell 1000 Defensive index. The long-term excess return objective is 300 - 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in July 2012.

The U.S. Low Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the Russell 1000 Index. The strategy targets modest above-market returns, with significantly lower standard deviation and a higher Sharpe Ratio than the index, over the long-term. The composite was created in August 2012.

The Eurozone Low Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI EMU Index. The strategy targets modest above-market returns, with significantly lower levels of standard deviation and a higher Sharpe Ratio than the index, over the long-term. The composite was created in August 2012.

The Emerging Markets Managed Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI Emerging Markets Index. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in May 2013.

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The Emerging Markets Low Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI Emerging Markets Index. The strategy targets modest above-market returns, with significantly lower levels of standard deviation and a higher Sharpe Ratio than the index, over the long-term. The composite was created in May 2013.

The U.S. Managed Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is the Russell 1000 Index. The strategy has a long-term target excess return objective of 300 – 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in June 2013.

The Global All Country Managed Volatility ex Australia strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI All Country World Index ex Australia. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in October 2013.

The Global Defensive Managed Volatility pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the Russell Global Large Cap Defensive Index. The long-term target excess return objective is 300 - 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in May 2014.

The Global Managed Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is MSCI World Index. The strategy has a long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in July 2014.

The Global Managed Volatility (EUR) strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly domestic large capitalization securities. The benchmark is MSCI World Index. The strategy has a long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in July 2014.

The Global All Country Managed Volatility strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The strategy pursues a risk-managed approach to construct a diversified portfolio of predominantly global large capitalization securities. The benchmark is the MSCI All Country World Index. The long-term target excess return objective is 300 – 400 basis points (annualized gross of fees). The strategy also targets a lower standard deviation and higher Sharpe Ratio than the index. The composite was created in October 2014.

The S&P 500 Index is an unmanaged index of 500 stocks that is generally representative of the performance of larger companies in the U.S.

For U.S. Large Cap Growth from inception to 12/31/05, the composite's benchmark was the S&P 500/Barra Growth Index ("Barra Growth Index"). In 2005, S&P announced index name and methodology changes affecting the Barra Growth Index, which later became the S&P 500/Citigroup Growth Index ("Citigroup Growth Index"). During the transitional period, from 1/1/06 to 3/31/06, the benchmark return consisted partially of the return of the Barra Growth Index and the Citigroup Growth Index. On 4/1/06 the composite's benchmark was changed to the Citigroup Growth Index. Effective 12/9/2009, the Citigroup Growth Index's name was changed to S&P 500 Growth Index.

The S&P 500 Growth Index is a market-capitalization-weighted index developed by Standard and Poor's consisting of those stocks within the S&P 500 Index that exhibit strong growth characteristics. The index measures the performance of the growth style of investing in large cap U.S. stocks. The S&P 500 Growth Index uses a numerical ranking system based on growth factors and value factors to determine the constituents and their weightings. The S&P 500 Index will be reconstituted annually.

The Russell 1000 Growth Index measures the performance of those Russell 1000 companies with higher price-to-book ratios and higher forecasted growth values.

The Russell 1000 Index measures performance of the 1,000 largest companies in the Russell 3000 Index.

The Russell 1000 Value Index measures the performance of those Russell 1000 companies with lower price-to-book ratios and lower forecasted growth values.

The Russell 1000 Defensive Index measures the performance of the large-cap defensive segment of the U.S. equity universe. It includes those Russell 1000 Index companies with relatively stable business conditions which are less sensitive to economic cycles, credit cycles, and market volatility based on their stability variables. Stability is measured in terms of volatility (price and earnings), leverage, and return on assets. The Russell 1000 Defensive Index is constructed to provide a comprehensive, unbiased and stable barometer for the large-cap defensive segment.

The Russell Global Large Cap Defensive Index measures the performance of the investable securities in the Global Large Cap Defensive segment of the market. The Russell Global Large Cap Defensive Index is constructed to provide a comprehensive and unbiased barometer for this market segment and is completely reconstituted annually to accurately reflect the changes in the market over time.

The MSCI USA Index is a free float-adjusted market capitalization-weighted index that covers approximately 85% of large cap and mid cap investable securities in the U.S. equity markets.

The MSCI North America Index is a free float-adjusted market capitalization-weighted index that is designed to measure the performance of the developed equity markets in Canada and the United States.

The MSCI World Index is a free float-adjusted market capitalization weighted index that is designed to measure global developed market equity performance.

The MSCI EAFE Index (Europe, Australasia, Far East) is a free float-adjusted market capitalization index that is designed to measure the equity market performance of developed markets, excluding the US & Canada.

The MSCI World ex Japan (Kokusai) Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of developed markets excluding Japan.

The MSCI Europe Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of the developed markets in Europe.

The MSCI All Country World Index is a free float-adjusted market capitalization weighted index that is designed to measure performance of global developed and emerging equity markets.

The MSCI World High Dividend Yield Index is a free float-adjusted market capitalization-weighted index based on the MSCI World Index, its parent index, which includes large- and mid-cap stocks across 24 Developed Market Countries. The MSCI World High Dividend Yield Index is designed to reflect the performance of equities with higher-than-average dividend yields and pass dividend sustainability and persistence screens.

The MSCI Emerging Markets Index is a free float-adjusted market capitalization-weighted index that is designed to measure performance of emerging equity markets.

The MSCI EMU (European Economic and Monetary Union) Index is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of countries within EMU.

The MSCI All Country World ex Australia Index is a free float-adjusted market capitalization-weighted index that is designed to measure performance of global developed and emerging equity markets excluding Australia.

The MSCI All Country World ex USA Index is a free float-adjusted market capitalization-weighted index that is designed to measure performance of global developed and emerging equity markets excluding the US.

The Index returns are provided to represent the investment environment existing during the time periods shown and are not covered by the report of independent verifiers. For comparison purposes, the index is fully invested, which includes the reinvestment of dividends and capital gains. The returns for the index do not include any transactions costs, management fees or other costs, and are gross of dividend tax withholdings unless otherwise noted. Composition of each separately managed account portfolio may differ from securities in the corresponding benchmark index. The index is used as a performance benchmark only, as INTECH does not attempt to replicate an index. The weightings of securities within the portfolio may differ significantly from the weighting within the index. The index is not available for direct investment; therefore, its performance does not reflect the expenses associated with the active management of an actual portfolio.

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With respect to the European Large Cap Core Composite, prices assigned to investments are published prices on their primary markets or exchanges since the composite's inception.

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The Power of Rebalancing: Fact, Fiction and Why it Matters

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It is well-understood that rebalancing is a necessary step in restoring a portfolio of volatile assets back to its target weights. Whether it is performed periodically or triggered when actual weightings move too far from target, rebalancing a portfolio will naturally lead to selling assets that have outperformed the portfolio, and buying assets that have underperformed the portfolio.

It is much less widely understood that rebalancing can actually be a source of return for the portfolio. Despite the fact that this observation dates back to 1982 [Fernholz and Shay] and has been successfully used to manage portfolios for nearly as long, it has come under considerable attack in the recent past by some academics and practitioners. The main arguments used by these detractors are:

1. There is no return benefit, because the portfolio's expected wealth does not increase.
2. The return benefit exists, but is due to diversification, not rebalancing.
3. The return benefit relies on mean-reversion.

These arguments may appear compelling at first glance, but all three are fundamentally flawed. Let's see why.

1. Compound returns are more relevant than expected returns.

The meaning of the expected return of a portfolio is often misunderstood. The expected return simply indicates how well the portfolio will perform, on average, in a *single predetermined* period of time. It is closely related to the concept of expected terminal wealth, which is the average value of the investment after a certain period of time (the 'term' of the investment). These measures can be useful for determining the soundness of an investment over a fixed period of time, but can also be highly misleading in the presence of compounding over multiple periods.

The real danger of focusing on expected return comes from overreliance on the simple average. Hundreds of years of research into statistics and probability theory have led to the understanding that the average does not tell the whole story. It is therefore perplexing to see papers published in the 21st century still claiming [Chambers and Zdanowitz; Cuthbertson et. al.; Qian] that two investments with the same expected return are equivalent (even if the lower-volatility investment is thereby acknowledged to be preferable). The real problem with this narrow view is that the *median* return can be much lower than the average return. This means that it is likely that the typical return experienced by the investor is substantially less than the expected return. Furthermore, compounding the investment over multiple periods usually exacerbates the difference between the median and the average, so that, after a number of iterations, it becomes less and less likely that the expected (i.e., average) return is achieved. Don't be fooled by the terminology: 'expected' doesn't mean 'likely.'

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To illustrate this, consider a speculative investment of $10 that has a 1% chance of hitting a $1,000,000 payoff in one year's time, but a 99% chance of falling to $1 (see Figure 1). The expected return of the investment is[1] a staggering 99,909.9%, but in this case the term 'expected return' is highly misleading: a rational person would probably 'expect' to lose $9 on this investment! It may still seem like a great opportunity since the upside potential is so huge, however very unlikely to be realized.

On the other hand, if you continue to reinvest your proceeds in the same investment, thus subjecting yourself to compounding, you will almost certainly be disappointed in the outcome. Assuming the different periods are independent, the 'expected return' becomes astronomically high over time, but there is also virtually no chance of getting anywhere near it. Indeed, for every five times the investment doesn't pay off, you need to hit the payoff once just to break even. (For example, even if it pays off the first time to give you the million, the next five losses will see you successively having $100K, $10K, $1K, $100 and then the original $10.) No surprise, then, that the compound return[2] of the investment is around -88.5%.

While the example is somewhat extreme in its outcomes, it is nonetheless illustrative of the main point: the expected return is most useful for understanding a reasonably narrow range of outcomes for a fixed term that is known in advance. For an investment with an open-ended time horizon, such as the vast majority of retirement-related investments, the compound return is what really matters. It is therefore appropriate to assess any gain in compound return as a return benefit, even if the expected return stays the same.

2. Rebalancing is vital to preserving the gains from diversification.

Suppose you now have access to a hundred uncorrelated investments of the type in the above example. With an initial capital of $1,000, it seems reasonable to place $10 in each of the hundred investments. In fact, the expected return is the same no matter what you do: it is the same staggering but unlikely 99,909.9% we saw in part 1 above, regardless of whether you diversify or put the whole $1,000 in one of the investments. Diversification has no impact on the expected average return. The *compound* return, however, does change. Instead of the awful −88.5% compound return for a single investment, the diversified portfolio has a compound return of 4,186%. This is because there is a greater than 63.3% chance that at least one of the hundred investments hits the big payoff.

If precisely one of the investments does pay off, an immediate rebalance is a great idea. Otherwise, almost all of the capital – $1M out of the $1.000099M, to be precise – remains in the successful investment, and the diversification you started out with has all but vanished. To keep the compound return high, not to mention actually get any compounding at that level of return, it is necessary to rebalance, redistributing most of the capital away from the previously successful investment to the other investments.

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[1] The expected return is computed as follows:

$$E(R) = 0.01 \times (1000000/10 - 1) + 0.99 \times (1/10 - 1) = 99{,}909.9\%.$$

[2] Whenever the term appears in this article, 'compound return' refers to the long-term compound return, in the limit as the number of periods tends to infinity. In many applications, for example when the returns are statistically independent, this equals $\exp(E(\log(1+R))) - 1$.

It is easy to dismiss rebalancing as simply a tool to preserve diversification, but the relationship between the two is almost symbiotic: you can't have one without the other. Without rebalancing, a portfolio generally tends to become more concentrated in the assets that have performed well, and it is only a matter of time before the diversification benefits erode away. Furthermore, the rebalancing trades themselves have a buy-low, sell-high quality and are profitable on average: in the above example, even though you buy 99 recent losers, the one winner that you sell more than makes up for those losses. The rebalancing has locked in the gains from the diversification, which otherwise might be lost.

Does diversification always improve the compound return? In general, it depends on how diversification is measured. Diversification as a tool for reducing risk has been studied for decades and several ways to measure it have been proposed over the years [Frahm and Wiechers; Chouifaty and Coignard], but only one method provides a direct measurement of the associated increase in compound return. This measure depends on the *difference between* (and not the *ratio of*) the average asset variance and the portfolio variance. To see why this is so, consider the well-known approximate relationship between expected return and compound return for a portfolio:

$$\text{compound return of portfolio} \approx \text{expected return of portfolio} - \tfrac{1}{2}\,\text{variance of portfolio} \qquad (1)$$

Half the variance of the portfolio is subtracted from the portfolio's expected return in order to arrive at the portfolio's compound return. As the variance of the portfolio appears to detract from the portfolio's compound return, the false conclusion is often drawn that if you reduce the portfolio's variance you will always increase the compound return. This is only true if the reduction in volatility does not also decrease the expected return of the portfolio.[3]

In the highly unlikely case where all the assets in a portfolio have identical expected returns, the expected return of the portfolio will indeed also be the same (in the absence of trading costs) regardless of the way you structure it. In this particular case, then, it is true that minimizing the variance will also give the highest compound return. It is easy, therefore, to make the mistake of concluding that diversification 'only' improves the compound return because volatility is decreased [Cuthbertson et. al.]. But if the expected returns of the individual assets in the portfolio are different, as is generally the case in the real world, one must apply the above relationship between compound and expected return individually to each asset:

$$\text{compound return of asset} \approx \text{expected return of asset} - \tfrac{1}{2}\,\text{variance of asset} \qquad (2)$$

Since the expected return of the portfolio is simply the weighted average of the expected returns of the assets in the portfolio, it follows that[4] there is a discrepancy between the *compound* return of a portfolio and the weighted average of the *compound* returns of the assets in the portfolio. This gap equals one half the difference between the average asset variance and the portfolio variance, and it represents the boost to the portfolio compound return arising from diversification:

$$\text{increase in portfolio compound return} \approx \tfrac{1}{2}\,(\text{weighted average asset variance} - \text{portfolio variance})$$

[3] In practice, reducing portfolio volatility without impacting expected return often requires portfolio optimization, which generally results in an increase in diversification.

[4] First, take weighted averages across all the assets in equation (2), then compare with equation (1). The first term on the right-hand side of each equation is the same (expected portfolio return equals weighted average expected return of assets). However, the second term is not, since the portfolio variance is less than the weighted average asset variance, if no short positions are allowed.

No other formula for diversification shows the return benefit this directly and generally. This formula also demonstrates that when the assets do not all have identical expected returns, minimizing the portfolio variance is *not* the optimal way to improve diversification: you have to take the individual asset variance into account as well.[5]

It immediately follows that, if the variances of the assets or the portfolio are changing over time, as they inevitably do in the real world, then the diversification benefit will also change. It would naturally make sense to respond to the changing landscape by modifying the asset weighting policy to take this into account. In that case, the trading necessary to maintain the portfolio is not, strictly speaking, 'rebalancing,' since the targets are not static, but are varying. Nonetheless, the terminology is still appropriate since the trading has the same character as traditional rebalancing: on average, you are systematically selling recent winners and reinvesting the profits in recent losers. This has the effect of locking in previous gains *and* keeping the portfolio more diversified, using the volatility of the individual assets within the portfolio to continue to push up the compound return of the portfolio as a whole.

3. Individual-asset mean-reversion is not necessary.

The above formulas are not always intuitive, even if the notion of buying low and selling high does sound reasonable. For this reason, it is common to demonstrate the effect of rebalancing by imagining two risky assets that are perfectly negatively correlated. For example, suppose that each asset has an even chance of rising by 25% or declining by 20% in a year, but due to the negative correlation, if one of the assets rises, then the other one falls. If asset A rises in the first year and falls in the second year, and asset B does the opposite, then the two-year return is zero for each of the two assets as well as for any buy-and-hold portfolio of the two assets. Figure 2(a) depicts a buy-and-hold portfolio that is initially equal-weighted, with $100 in each asset.

Rebalancing to equal weights produces a different situation: by selling the recent winner and investing in the recent loser, the portfolio returns 2.5% per year, as is depicted in Figure 2(b). In fact, so long as the portfolio is always rebalanced back to equal weights, then no matter which of asset A and asset B happens to rise, the portfolio always returns 2.5% a year.

Even though Figures 2(a) and 2(b) appear to show mean-reverting assets, the certain 2.5% annual return in the case of rebalancing to equal weights does not depend on this. This is clear in the case shown in Figure 3(b): if asset A rises twice and asset B falls twice, the portfolio value still rises from $205 to $210.125 at the second step: a return of 2.5% a year. This case may be thought of as the 'trending,' or non-mean-reverting, case.

How about the non-rebalanced (i.e., buy-and-hold) portfolio? To understand this, it is necessary to look at the two cases separately. In the reversion case (Figure 2), the $200 initial investment ends up at $200, so the rebalanced portfolio is superior. In the trending case (Figure 3(a)), the buy-and-hold portfolio ends up at $220.25, outperforming the rebalancing case. Since these two outcomes are equally likely in this example, and the average is the same whether or not one rebalances,[6] have

[5] If all the assets are identical, then the weighted average asset variance is the same no matter what the weighting. In this special case, the only way to increase diversification is evidently to reduce the variance.
[6] Rebalancing always produces $210.125 after the second step; the buy-and-hold either produces $200 or $220.25 with equal probability; the average of $200 and $220.25 is indeed $210.125.

we gained anything by rebalancing, other than reducing the volatility by narrowing the outcomes from two to one?

Yes! The compound return of the rebalanced portfolio has improved relative to the buy-and-hold one. In the buy-and-hold case, the compound return in the second year is 2.38%, as opposed to 2.5% for the first year.[7] This comes from the drop in diversification, which inevitably follows the failure to rebalance. If this continues for 100 years, the expected terminal wealth is $2,362.74, but the realized wealth is less than this more than 86.6% of the time. The compound return for the entire period is only about 1.24%, compared to a steady 2.5% for the rebalanced portfolio.

This analysis did not assume any mean-reversion of asset prices. Every possibility was included, including the highly unlikely 'ultimate trending' case where one of the assets rises each year for 100 years.[8] The expected terminal wealth is rarely achieved, but, when it is, the actual payoff is often huge due to the concentration of wealth in one of the assets. It is true that systematic rebalancing would have prevented this rare opportunity from transpiring. It is worth noting that if one excludes outcomes where one of the two stocks ends up with a market weight greater than 99.99%, the expected terminal wealth of the buy-and-hold portfolio drops to $1,122.18, while the rebalanced portfolio still ends up at $2,362.74. Once again, the notion of expected terminal wealth fails to describe the likely outcome.

Any two-asset example should be thought of as illustrative only. One may retrospectively find a case of two stocks that exhibit mean-reverting behavior [Bouchey et. al.], or alternatively a case where one stock dominates the other; these examples cannot possibly prove or disprove anything about the diversification/rebalancing effect any more than a run of five heads in consecutive coin flips 'proves' that the coin is biased.

Real-world portfolios – especially stock portfolios – are usually comprised of many more than two assets. The correlations and variances of these assets will vary over time, but the benefits of diversification and rebalancing persist. If there is extreme concentration in a handful of assets that ends up dominating the others, a buy-and-hold approach could well have worked out better in retrospect. In other circumstances, where no group of assets dominates for long periods, even if none of the individual asset prices are mean-reverting, the collective stability of the market will tend to favor the rebalancing approach.[9] While analysis of this sort of stability is beyond the scope of this article, there is a rich body of literature that indicates that equity markets generally do behave in a stable manner, where no one stock or group of stocks persistently dominates, even though most of the stock prices are not individually mean-reverting. These are, in fact, ideal circumstances for a rebalancing approach to deliver results.

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[7] The computation is straightforward: since the first period ends with $205, and the second period ends with an equal chance of $200 or $220.25,

$$\text{compound return} = \exp(\tfrac{1}{2}\log(200/205) + \tfrac{1}{2}\log(220.25/205)) - 1 \approx 2.38\%.$$

[8] A corroborating spreadsheet is available upon request from the authors. The experiment can also be analyzed using Monte Carlo simulations, but this is unnecessary since an explicit mathematical analysis is straightforward.

[9] Care must be taken to estimate transaction costs for the scale of assets being traded, then select a trading strategy so that these costs do not overwhelm the expected benefit.

Conclusion

Compound returns are a vital attribute for any investment whose term length is not known in advance, since they are more reflective of *typical* outcomes. Even if compound returns for each asset are hard to predict in advance, maintaining a diversified portfolio by means of rebalancing increases the compound return of the portfolio as a whole relative to a buy-and-hold portfolio. The formula for this benefit, originally identified by INTECH's founder, Dr. E. Robert Fernholz in 1982, specifies the amount of this increased return to the extent that volatilities can be estimated, gross of transaction costs. Rebalancing and diversification are inextricably linked concepts. Mean-reversion of individual asset prices is absolutely not required for this phenomenon to hold, and stability of the overall market structure further reduces the likelihood that a buy-and-hold approach will outperform the diversified/rebalanced approach in the long term.

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Figure 1

Figure 2: Performance of Portfolio, mean-reverting case: (a) buy-and-hold portfolio (b) rebalanced portfolio

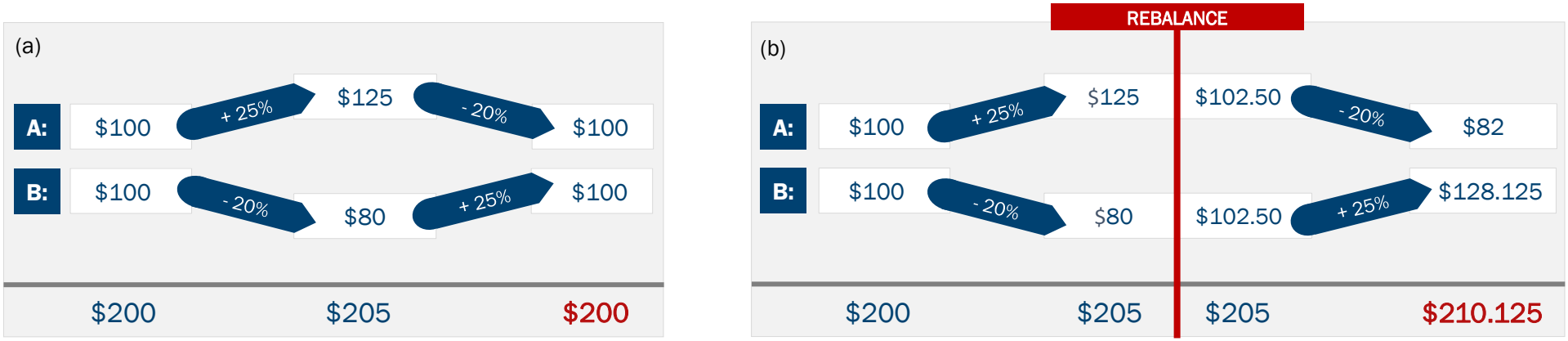
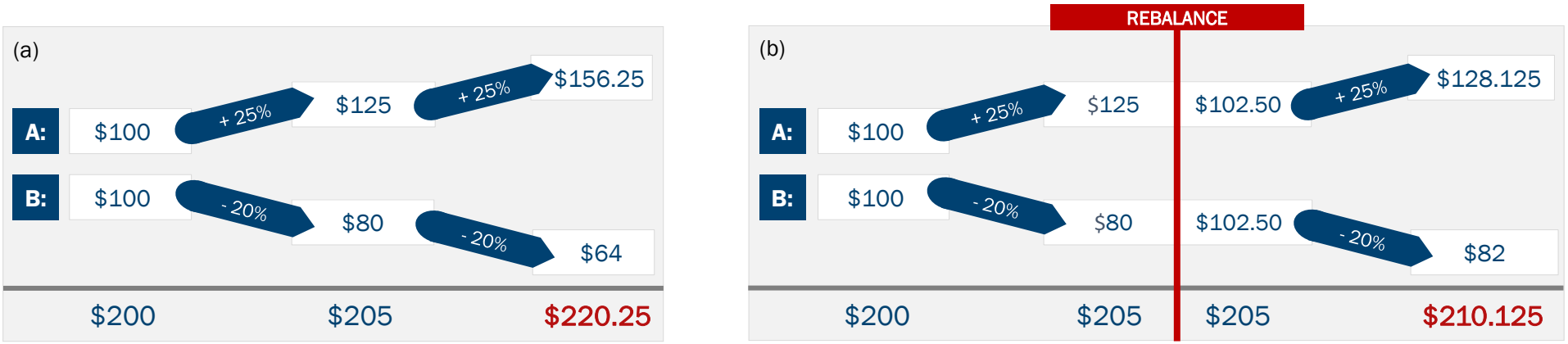


Figure 3: Performance of Portfolio, trending case: (a) buy-and-hold portfolio (b) rebalanced portfolio



A Note on the Sources of Portfolio Returns: Underlying Stock Returns and the Excess Growth Rate*

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A Note on the Sources of Portfolio Returns: Underlying Stock Returns and the Excess Growth Rate

Abstract

A portfolio's compound return over time is not simply the weighted sum of the compound returns of its underlying stocks. Instead, it is due to (A) the underlying constituent stocks' compound returns, and (B) a component induced by constituent covariances. This can be important. The average smallest-cap decile portfolio outperformed its largest-cap counterpart by 44bps/mo, but the smallest-cap decile stock constituents on average *underperformed* their largest-cap counterparts by 74bps/mo. Thus, the "size effect" is not a small-firm effect, but a small-firm *portfolio* effect. In contrast, our HML and UMD portfolios outperformed because their individual stock constituents outperformed on average. Value and momentum are simultaneously portfolio and individual stock effects.

1. Introduction

It is tempting to assume that the performance of a portfolio is equivalent to the performance of its underlying stocks. Unfortunately, this assumption can lead to incorrect inferences regarding the portfolio's constituents and can obscure richer observations about a portfolio's performance and that of its underlying stocks. This paper focuses on two distinct sources of portfolio performance. The first source arises from the familiar first moment of the portfolio's underlying stocks' returns[1]. The second source, though not a second order effect, arises from the variances and covariances of the portfolio's underlying stocks. We empirically estimate these sources based on the Fernholz and Shay (1982) mathematical model of portfolio returns.

The following simple example illustrates the underlying economic issue that is the focus of this paper. Suppose there are four stocks with returns over two periods as given in Exhibit 1. First, consider stocks A_1 and A_2. Each stock doubles in value one period and loses half its value in the other period. Both stocks will have a 25% average raw return over the two periods, but an average log return of zero. Even though the average log return is zero for each stock, an equal-weighted portfolio (EW_A) of the two stocks earns a 22.31% log return per period.

Now consider stocks B_1 and B_2, which both have raw returns of 25% (log returns of 22.31%) each period. An equally-weighted portfolio (EW_B) of these two stocks will have an arithmetic return of 25% (log return of 22.31%) per period. Thus, portfolio EW_B performs

[1] We adopt the same terminology as Campbell, Lo, and MacKinlay (1998) when referring to returns. Unless otherwise indicated, "return" refers to the continuously compounded return on a stock or portfolio. This may be presented as the continuous return, log return, geometric return, compound return, or growth rate in other contexts and captures the average growth rate of a stock or portfolio over multiple time periods. In an effort to be clear, we sometimes emphasize the distinction from simple (i.e., raw or arithmetic) returns with explicit references to compound returns and raw returns.

identically to portfolio EW_A, but the sources of the returns are strikingly different. In portfolio EW_A the return is driven by the variance of the underlying stocks. However, the return of portfolio EW_B is a direct result of the log returns of the two stocks. We provide a method for parsing portfolio returns into these two distinct sources.

Exhibit 1

	Raw Returns				Log Returns			
	Period 1	Period 2	Average	Variance	Period 1	Period 2	Average	Variance
A_1	+100%	-50%	+25%	0.5625	+0.6931	-0.6931	0.0000	0.4805
A_2	-50%	+100%	+25%	0.5625	-0.6931	+0.6931	0.0000	0.4805
EW_A	**+25%**	**+25%**	**+25%**	**0.0000**	**+0.2231**	**+0.2231**	**+0.2231**	**0.0000**
B_1	+25%	+25%	+25%	0.0000	+0.2231	+0.2231	+0.2231	0.0000
B_2	+25%	+25%	+25%	0.0000	+0.2231	+0.2231	+0.2231	0.0000
EW_B	**+25%**	**+25%**	**+25%**	**0.0000**	**+0.2231**	**+0.2231**	**+0.2231**	**0.0000**

Our analysis primarily concerns the cross-sectional aggregation of individual stock returns to portfolio returns. This concept is reflected in Exhibit 1 by the fact that a portfolio's log return does not necessarily equal the average log return of its constituent stocks. In the case EW_A, the average log return for the portfolio is 22.31%, while the average log return of its constituent stocks is zero. This phenomenon is distinct from the more widely recognized fact that average log returns for an individual stock over time do not simply equal the log of the average raw returns for that stock over time. For example, the average log return for stock A_1 is zero, but the log of the average raw return of 25% is 22.31% for A_1.

Fernholz and Shay (1982), hereafter FS, offer what we believe to be the first rigorous mathematical analysis to identify the sources of the long-term performance of a portfolio. Specifically, FS show that a portfolio that is rebalanced to the same constant weights has a compound return that is a function of the underlying stocks' compound returns and an "excess

growth rate" that is due to the difference between the stocks' variances and the (diversified) portfolio's variance.[2] This dichotomy implies that a portfolio's return can be increased by 1) choosing stocks that have higher returns than their peers; and/or 2) choosing a more favorable mix of stocks based on their variances and covariances. The former component arises from differences among stocks' cross-sectional returns, while the latter component arises from differences among stocks' volatilities and covariances.

We illustrate the importance of the FS model by empirically estimating the sources of portfolios' returns for several characteristic-based portfolios that appear frequently in the literature. Our results reveal that a portfolio of stocks from the smallest market cap decile have average monthly compound returns of 44 bps per month more than large stock portfolio, but the average compound returns of stocks in the small-cap portfolio are 74 bps lower than the returns of stocks in the large-cap portfolio. The difference of about 118 bps between these two estimates derives from the excess growth rate that is induced by the higher variance of stocks in the small-cap portfolio. When sorted by book-to-market ratios, the highest book-to-market portfolio has portfolio monthly compound returns that are 127 bps higher than the lowest book-to-market portfolio. In contrast to size portfolios, the difference in portfolio compound returns is driven primarily by the difference in the average underlying stock compound returns for book-to-market portfolios, as the highest book-to-market portfolio's stocks have an average compound return that is 157 bps higher than average compound return for the stocks in the lowest book-to-market portfolio. Portfolios based on past returns and estimates of beta display both weaker and

[2] Brennan and Schwartz (1985) independently offer a similar derivation of what they term a positive "bias" in the return of an equally weighted portfolio (a special case of the FS analysis) over and above the return of an equally weighted geometric index. Booth and Fama (1992) separately derive a similar decomposition of a portfolio's return and attribute a "diversification return" to an asset's contribution to the portfolio return that is in excess of the asset's compound return. Booth and Fama (1992) provide some analysis at the asset allocation level across indexes, but do not examine the potential effects that occur within portfolios across stocks.

more non-linear patterns across decile portfolios, yet they also exhibit meaningful differences between portfolio returns and underlying average stock returns.

Our parsing reveals interesting new insights into the sources of portfolio returns in general, and the sources of returns to oft-studied characteristic-based portfolios, such as size (Banz, 1981), market-to-book (Fama and French, 1992), and momentum (Carhart, 1997) portfolios, in particular. For example, the outperformance of small firm portfolios can be attributed to the influence of the constituent stocks' variances on the portfolio's return, not to the returns of the underlying stocks themselves. Our key results illustrate this with the seemingly paradoxical result that *portfolios* of small stocks outperform, while small stocks underperform, on average. The apparent paradox of having outperforming portfolios comprised of underperforming stocks is easily resolved in the mathematical parsing of portfolio returns that we present herein.

2. Background

Numerous academic studies have demonstrated how cross-sectional variation in the characteristics of stocks is related to portfolio performance over time (Brown and Warner, 1980; Barber and Lyon, 1997; Kothari and Warner, 1997; Lyon, Barber and Tsai, 1999; Boynton and Oppenheimer, 2006; Albuquerque, 2012). However, this generalization from individual stocks to portfolios and vice versa is subject to inherent limitations as the time-series patterns of individual stock returns do not directly aggregate to portfolio return patterns. We therefore approach the issue from the opposite direction. By imposing precise and consistent measurement methods on stock and portfolio returns by using continuously compounded (i.e., log) returns and fixed rebalancing frequencies, we are able to parse portfolio returns into the underlying contributions

made by the constituent stocks. This allows us to undertake an analysis of the cross-sectional characteristics of individual stocks and their contribution to a portfolio's return that was not utilized in past studies.

Firm size is a good example of a cross-sectional stock-level characteristic that can be used to show how economic explanations can be greatly refined through our decomposition of portfolio returns. Discussions of the size effect typically fall into two categories: 1) portfolio comparisons: "Portfolios of small firms outperform portfolios of large firms"; and 2) stock comparisons: "Small firms earn higher returns than large firms." For example, Bodie, Kane, and Marcus (*Investments*, 9th ed., p. 361) fall into the former case, noting that "Average annual returns between 1926 and 2006 are consistently higher on small-firm portfolios." However, others often provide the latter explanation. For example, Reilly and Brown (*Investment Analysis and Portfolio Management*, 8th ed., p. 180) note in reference to studies of the size effect, "During most periods they found the negative relationship between size and return; but, during others (such as 1967 to 1975), they found that large firms outperformed the small firms." Similarly, Sharpe (*Investors and Markets*, 1st ed.) notes, "small stocks seem to have performed better than large stocks"; and Malkiel (*A Random Walk Down Wall Street*, 1999, p. 250) describes the size effect as, "the tendency over long periods of time for small stocks to do better than large stocks." An often overlooked problem here is that, with compound returns, one category does not imply the other. Indeed, it is possible to have a portfolio of stocks that outperform over the long term, even though the underlying stocks are below-average performers.

To decompose a portfolio's return, we begin with the returns of the portfolio's underlying assets. Let $\gamma_{i,t}(h)$ be the compound return (i.e., continuously compounded or log return) for stock *i* at time *t* over a holding period of length *h*. If $\gamma_{i,t}(h)$ is normally distributed (i.e., raw stock

returns are log-normally distributed) with mean $\bar{\gamma}_i(h)$ and variance $\sigma_i^2(h)$, then the average compound return and the average holding period return, $\bar{r}_i(h)$, are related to one another as

$$\bar{r}_i(h) \approx \bar{\gamma}_i(h) + \frac{\sigma_i^2(h)}{2}.^{3} \tag{1}$$

If stock holding period returns are not log-normal then it is an empirical question whether (1) is a useful approximation. If stock returns are log-normally distributed, then portfolio returns are not log-normally distributed. Similarly, if portfolio returns are log-normally distributed, the average holding period return of portfolio *p* can be expressed analogously as for individual stocks:

$$\bar{r}_p(h) \approx \bar{\gamma}_p(h) + \frac{\sigma_p^2(h)}{2}. \tag{1'}$$

Using the definition of the arithmetic average return on a portfolio and Eq. (1), the arithmetic average return on a portfolio that maintains a constant weight in each stock is given by

$$\begin{aligned} \bar{r}_p(h) &= \sum_{i=1}^{N} w_i \bar{r}_i(h) \\ &= \sum_{i=1}^{N} w_i \bar{\gamma}_i(h) + \frac{1}{2}\sum_{i=1}^{N} w_i \sigma_i^2(h), \end{aligned} \tag{2}$$

where w_i is the portfolio weight in stock *i*. Note that this portfolio is implicitly rebalanced every holding period *h* to maintain the constant weights. Combining (2) and (1), the average compound return on a portfolio is

$$\bar{\gamma}_p(h) = \sum_{i=1}^{N} w_i \bar{\gamma}_i(h) + \frac{1}{2}\left[\sum_{i=1}^{N} w_i \sigma_i^2(h) - \sigma_p^2(h)\right]. \tag{3}$$

[3] The exact relationship is $\ln[1+\bar{r}_i(h)] = \bar{\gamma}_i(h) + \sigma_i^2(h)/2$. We use the approximation formula for ease of exposition and illustration, ignoring the approximation beyond Eq. (2) and assuming it holds exactly. The empirical validity of this assumption and approximation are confirmed by the results herein.

Using continuous rebalancing, Fernholz and Shay (1982) derive a continuous-time version of (3) and identify the second term as a portfolio's *excess growth rate*. We denote the discrete time portfolio excess growth rate as

$$\gamma_p^*(h) = \frac{1}{2}\left[\sum_{i=1}^{N} w_i \sigma_i^2(h) - \sigma_p^2(h)\right]. \tag{4}$$

The purpose in identifying the holding period length *h* in the discrete time version is to emphasize the importance of matching the measurement interval of the variances and covariances with the frequency of rebalancing to the constant weights. For example, when using monthly returns to estimate Eq. (3), monthly rebalancing is implied.

Equation (3) shows that a portfolio's compound return comes from two primary sources: 1) the weighted average compound returns of the constituent stocks; and 2) the portfolio's excess growth rate. Not surprisingly, weighting stocks with high returns benefits a portfolio's return. Less obvious is the impact to a portfolio's return that arises from the excess growth rate. Specifically, any portfolio that heavily weights stocks with relatively high variances should benefit, assuming that the stocks are not perfectly correlated so that the portfolio's variance is not increased proportionally.[4] Potential candidates for portfolios that put relatively high weight on high-variance stocks are equally-weighted portfolios, contrarian portfolios, or small stock portfolios. To the extent that the stocks in these portfolios have higher variances, they should have higher excess growth rates.

Equation (3) is useful beyond its implications for portfolio analysis. Specifically, many studies utilize, either explicitly or implicitly, constant-weight portfolios in their analysis. Indeed,

[4] Booth and Fama (1992) derive an analogous expression for a portfolio's average log return from the Taylor series expansion of the natural logarithm, referring to the excess growth effect as the "diversification return." Their analysis focuses on the diversification return to various asset-class-level indexes and does not consider the effect within the indexes and asset classes that they consider.

any study employing panel data that averages the returns across stocks and then considers the average of this measure over time is subject to these effects that act on a portfolio's return. The return of Portfolio A could exceed the return of Portfolio B if: 1) the stocks in Portfolio A have higher returns than the stocks in portfolio B; *or* 2) the excess growth rate of Portfolio A exceeds that of Portfolio B; *or* both. It would be inadvisable to infer from a higher return of Portfolio A compared to Portfolio B that the stocks in Portfolio A have higher returns compared to stocks in Portfolio B just by considering the portfolios' returns. Equation (3) allows researchers to parse the effects and distinguish between an interpretation that implies something about the *underlying firms* and another interpretation that implies something only about *portfolios* of such firms. In some circumstances, this subtlety might be considered negligible semantics, while in other circumstances it might be critical to the interpretation and implications of the study.

3. The Excess Growth Rates of Factor-based Portfolios

Fama and French (1992) (hereafter, FF), among others, show a "size" effect in which portfolios of small (Low market equity (ME)) stocks have higher average returns compared to portfolios of large (High ME) stocks, as well as a book-to-market effect where portfolios of high book-to-market (High BE/ME) stocks have higher average returns compared to portfolios of low book-to-market (Low BE/ME) stocks. Carhart (1997) incorporates the momentum effect of Jegadeesh and Titman (1993) into FF's model and shows that past performance can explain cross-sectional differences in the portfolios of mutual funds. While the momentum effect implies that current performance is positively related to (short-term) past performance, earlier work by DeBondt and Thaler (1987) suggests an opposite pattern of "reversals" over longer horizons.

The analysis below shows that the size and reversal effects are driven by the relative excess growth rates of portfolios sorted by size or long-term past performance. In contrast, the book-to-market and short-term momentum effects are actual characteristics of the underlying stock returns. Specifically, small stock and long-term reversal portfolios have higher returns because small stocks and stocks that have performed poorly over the previous 3-5 year period have higher variances, on average, leading to higher excess growth rates for portfolios composed of these stocks. These portfolios have higher returns despite the fact that small and contrarian stocks have lower compound returns, on average. In other words, these stocks do not outperform on average, but portfolios composed of them do. In contrast, high book-to-market portfolios and short-term momentum portfolios outperform because the constituent stocks have higher average compound returns.

For the sample period of January 1960 through December 2012, we match monthly raw return and capitalization (ME) data for non-financial common stocks from CRSP with book value of equity (BE) data in Compustat. We construct size, book-to-market and beta decile portfolios in a similar manner as in FF[5]. As in FF, the ME and beta decile breakpoints are determined using only NYSE stocks, while stocks from NYSE, AMEX, and Nasdaq are used to determine decile breakpoints for book-to-market portfolios. To analyze momentum and reversal effects, we sort stocks based on their returns in period *t-1*. We explore a wide range of prior return horizons, ranging from 1 to 60 months, in 3-month intervals, but we limit our reported results to portfolios formed based on prior 6-month and 5-year periods.

[5] We find that using calendar-year returns give almost identical results to our analysis as using July-to-June returns and we therefore report results based on calendar-year returns.

Once our characteristic-based portfolios are formed, equally weighted average portfolio compound returns are calculated for each month of year t[6]. As in the previous section, the natural log of (one plus) a stock's raw return is used to calculate the stock's compound return. Similarly, a month's actual portfolio compound return is the natural log of (one plus) the portfolio's raw return in that month. Annual variances are calculated using monthly log returns.

3.1. Size Decile Portfolios

Panel A of Table 1 shows the average raw returns, compound returns, and the estimated components of returns from Eq. (3) for the size (*ME*) decile portfolios. The first column of estimates contains the arithmetic average portfolio raw return, $\bar{r}_p$, over the sample period. The pattern of arithmetic average raw returns across size deciles is similar to that in Fama-French (1992).[7] The average return of the small stock (Low *ME*) portfolios is generally higher than the average return of the large stock (High *ME*) portfolios. While widely known, it is important to note that the true growth in an investor's wealth as given by the average actual compound return, $\bar{\gamma}_p(act.)$, when holding one of these portfolios is not reflected accurately by the arithmetic average raw return of the portfolio's constituent stocks. While also possibly known, but almost never explicitly acknowledged, the average portfolio compound returns, $\bar{\gamma}_p(act.)$, are poor estimates of the average compound returns of the portfolio's underlying assets, $\sum_{i=1}^{N} w_i \bar{\gamma}_i$.

The pattern in average portfolio compound returns, $\bar{\gamma}_p(act.)$, is similar to the pattern in average portfolio raw returns, $\bar{r}_p$. Likewise, the pattern in estimated average portfolio compound

[6] Although our reported results focus on average monthly returns over year *t*, we also examine, but do not report, results over both longer and shorter performance evaluation periods. We find that our portfolio return decomposition continues to provide insights into the sources of portfolio returns over these alternative periods, although a detailed analysis of this would be beyond the scope of the current study.

[7] We have also conducted our analysis using the FF sample period of 1963 through 1990. We obtain results that are nearly identical to those for our 53 year sample period.

returns, $\bar{\gamma}_p(est.)$, which is based constituent stocks' average compound returns and variances and portfolio variance in Eq. (3), conforms quite well to the pattern in actual portfolio compound returns. However, the average compound return to the underlying assets, $\sum_{i=1}^{N} w_i \bar{\gamma}_i$, reveals a pattern that is strikingly different than that of the average portfolio returns. Specifically, smaller stocks have lower average compound returns than larger stocks. The excess growth rate reconciles this apparent contradiction. The decomposition using Eq. (3) and Eq. (4) shows that the outperformance of the portfolios of smaller stocks is driven by the excess growth rate, $\bar{\gamma}_p^*(est.)$, of those portfolios.

The average portfolio excess growth rates decrease almost monotonically as the size decile increases from the smallest to the largest stocks. The same pattern appears in the estimated portfolio return that is calculated from the components of Eq. (3). The estimated average portfolio return, $\bar{\gamma}_p(est.)$, in the third column corresponds well to the actual average portfolio return, $\bar{\gamma}_p(act.)$, in the second column.[8]

The decrease in portfolio returns as the size decile increases occurs despite the fact that the underlying average stock compound returns increase as the size decile increases. Thus, the driving factor in the outperformance of small stock portfolios is the variance of the underlying stocks. For the entire sample period, the smallest size decile portfolio holds stocks with an average variance of 0.0308 per month, while the largest size decile portfolio holds stocks with an average variance of only 0.0067 per month. The variance of the small stocks is enough to generate portfolio excess growth rates that make up for the small stocks' lower returns. The

[8] We have also validated, but do not report, the empirical decomposition of Eq. (3) by estimating the portfolio return from the equation's components and comparing it to the actual portfolio return each year. A regression of the yearly estimated returns on the returns for an equally weighted portfolio of CRSP stocks yields a slope coefficient estimate of 1.0, a statistically insignificant intercept coefficient, and an R-squared exceeding 0.999.

decline in the portfolio returns as the size of the underlying stocks increases is driven by decline in the portfolios' excess growth rates due to the decline in the underlying stocks' variances.

3.2. Book-to-Market Portfolios

Panel B of Table 1 shows the average returns and the estimated components from Eq. (3) for portfolios formed using book-to-market ratio (*BE/ME*) deciles. Again, these results are nearly identical to those of FF, in that the lowest deciles have the lowest average performance. In contrast to the results for the size decile portfolios, the pattern of average portfolio returns across book-to-market deciles appears to be driven primarily by the underlying stocks' average compound returns. That is, the average returns of low *BE/ME* stocks are lower than the returns of high *BE/ME* stocks.

The average stock returns increase monotonically from the lowest *BE/ME* decile to the highest *BE/ME* decile. The spread between the highest and lowest *BE/ME* deciles' average stock returns is 1.57%, while the spread between the highest and lowest size deciles is only 0.74% (and in the opposite direction as expected in that large stocks have higher average compound returns than small stocks).

As in the size decile results, the average portfolio excess growth rates decrease monotonically as the *BE/ME* decile increases because the variance of the portfolios' underlying stocks decrease monotonically, except for the highest *BE/ME* decile. Across the entire sample period, the lowest *BE/ME* decile portfolio's excess growth rate is 1.36% compared to the highest *BE/ME* decile portfolio's excess growth rate of only 1.14%. However, this decrease is smaller in magnitude than in the size deciles and is more than offset by the increase in the underlying stocks' average returns. Unlike size decile portfolios, the driving factor in the outperformance of low *BE/ME* stock portfolios is the average compound returns of the underlying stocks.

3.3. Beta Portfolios

When stocks are sorted by their (lagged) beta estimates, we find no relationship between beta decile rankings and portfolio returns. This is consistent with FF. Panel C of Table 1 presents the components of returns based on deciles formed from beta, as estimated over the previous 5-year period using the CRSP VW index. Although portfolio returns are unrelated to beta, average stock returns show a non-linear and negative association with beta. Contrary to the traditional CAPM, we find that higher beta estimates are generally associated with lower average stock returns. The negative association between average stock returns and beta is almost perfectly balanced by a positive association between excess growth rates and beta, resulting in essentially no correlation between beta and portfolio returns.

3.4 Portfolios formed based on past performance

Existing studies of the persistence of past performance are complicated by several considerations. Returns at very short horizons (i.e., less than one month) may display substantial microstructure effects and other biases. At short to medium horizons, a momentum effect has been documented where top-performing portfolios in period *t-1* have above-average performance in period *t*. At longer horizons (i.e., 3 to 5 years) a reversal, or contrarian, effect appears to dominate, with portfolios of the worst performing stocks from period *t-1* performing relatively better in period *t*. Table 1 displays the components of monthly returns over year *t* for portfolios formed from performance during prior horizons of 6-months (Panel D) and 5 years (Panel E).

We find that the momentum effect is a robust empirical characteristic of average stock returns ($\sum_{i=1}^{N} w_i \bar{\gamma}_i$) in year *t*, regardless of the prior return horizon used to form portfolios. That is, if a portfolio's stocks had high returns in the prior period then that portfolio's stocks tend to have high returns, on average, in the current year. Although the relationship between average stock

returns and prior performance is generally upward-sloping, the pattern is non-linear. The pattern in average compound stock returns increases monotonically at first, peaking at decile 7, and then decreases slightly for the remaining deciles.

The momentum effect that we find for portfolios formed from past 6-month performance periods (consistent with existing papers such as Jegadeesh and Titman, (1993; 2001)) is also present for portfolios formed from long-term (5 year) past performance periods (apparently contrary to the reversal effect documented by DeBondt and Thaler (1985; 1987)). In untabulated tests, we find this result to be robust to a wide range of portfolio formation horizons ranging from 1-month to 5 years. That is, a momentum effect in underlying average stock returns (over a current period of 1-year) is a pervasive effect across all portfolio formation horizons. One important distinction here is that the results mentioned above apply to the average compound returns ($\sum_{i=1}^{N} w_i \bar{\gamma}_i$) of the stocks in each portfolio, while existing research generally examines portfolio returns ($\bar{r}_p$ or $\bar{\gamma}_p$).

Consistent with the published literature, at the portfolio level we also observe momentum effects for portfolio returns ($\bar{r}_p$) only when portfolios are composed based on past return horizons of 2 years or less. When portfolios are composed based on prior performance periods of longer than 2 years, then the excess growth effect dominates over the momentum effect present in the returns ($\sum_{i=1}^{N} w_i \bar{\gamma}_i$) of the underlying stocks for the stocks that performed poorly in period *t-1* and we therefore observe a reversal pattern in portfolio returns. More precisely, the portfolios composed of stocks with low returns in the prior 2-to-5 year period have sufficiently large excess growth rates ($\bar{\gamma}_p^*$) arising from the variance of the those stocks ($\sum_{i=1}^{N} w_i \sigma_i^2$) so that the

current portfolio return ($\bar{\gamma}_p$) is high enough to display a "reversal" effect, even though the average stock returns ($\sum_{i=1}^{N} w_i \bar{\gamma}_i$) of the portfolio's constituents are still relatively low on average.

Overall, we find that the momentum effect can be generalized to underlying stock returns only when portfolios are formed based on prior return horizons of 2 years or less. The contrarian, or reversal, effect for portfolios formed based on long-term (2 years or greater) prior performance is only a portfolio-level effect that cannot be generalized to the underlying stocks.

4. The Cross-Section of Stock Returns

Beginning with Fama and Macbeth (1973), cross-sectional regressions have been used to examine the association between stock returns and stock characteristics. Cross-sectional analysis appears to have the desirable property of ease of interpretation in that the slope coefficients from cross-sectional regressions reflect a direct association between a firm's return (dependent variable) and the firm's characteristics (independent variables). The attribution of the portfolio's return in the previous sections shows that a difference can exist between the returns to a portfolio that is formed on a specific stock characteristic and the average return on the underlying stocks with that characteristic. To further examine the role of stocks' variances in explaining the association between a stock's return and its other characteristics, we estimate Fama-MacBeth-style cross-sectional regressions for both raw and compound (log) stock returns.

The results in the previous section raise questions about the commonly accepted view that smaller firms have higher returns. To further examine this, we include the same-year variance of a stock's monthly returns (*Variance*) as an independent variable in cross-sectional regressions, in addition to using *ln(ME)* and *ln(BE/ME)* to measure a firm's size and market-to-book ratio, respectively. We include a stock's *beta* estimate, using the pre-ranking *beta* estimate

from FF, as a control variable. If the size effect in portfolio returns is due to stock variances, then *ln(ME)* should not be negatively associated with returns when *Variance* is included in the regression. More importantly, based on the result in the previous section that average stock returns are higher for firms in the larger size deciles, we expect the average slope on *ln(ME)* to be positive when using log returns as the dependent variable.

We report the time-series average slope coefficients from monthly cross-sectional regressions in Table 2. As shown in Panel A, the average slope on *ln(ME)* in the monthly raw return univariate regressions is negative over the period from January 1960 through December 2012.[9] The average coefficient on *ln(BE/ME)* is statistically significant and positive. *Variance* also takes a significant and positive coefficient estimate, while *beta* is unrelated to raw returns. In the multivariate regressions with all four explanatory variables, the average coefficients on *ln(ME)* and *beta* are insignificant, while the average coefficients on *ln(BE/ME)* and *Variance* remain positive and statistically significant.

Our return decomposition and cross-sectional regressions show that the volatility of small stocks is the primary contributor to small stock portfolios' returns, suggesting that caution should be exercised when inferring any long-term return premium on a stock due to *ln(ME)*. With the average cross-sectional correlation between *ln(ME)* and *Variance* of log returns being -0.46, the effect of multicollinearity in the cross-sectional regressions must be considered. We note that the average slope coefficient on *Variance* remains relatively stable when *ln(ME)* is added to the model. Moreover, the results using stocks' log returns as the dependent variable in regressions casts more doubt on a size effect in individual stock returns. Over the entire sample period, the coefficient on *ln(ME)* in monthly stock log return regressions is, on average, *positive* and

[9] We have estimated the cross-sectional regression analysis using the FF sample period of 1963-1990 and obtain similar results.

statistically significant. The average coefficient remains positive but statistically insignificant in multivariate regressions. The average coefficient for *ln(BE/ME)* remains positive and significant, while the average coefficient estimate for *beta* is insignificantly negative in multivariate regressions. The average slope on stocks' own variances is statistically insignificant and negative in explaining log returns.

We conduct a *J* test (Davidson and MacKinnon, 1981) to determine whether size or variance appears to be the true underlying factor in the cross-section of monthly raw stock returns. In the first stage of this test, returns are regressed on variance (log size) and the resulting coefficient estimates are used to compute a predicted value of returns based on information from variance (size). In the first row of Panel B, actual returns are regressed on the actual values of size and the predicted values of returns when variance is used to forecast returns. The second row of Panel B reports estimates from a model using the actual values of variance and the size-predicted values of returns. The first row indicates that size has no additional power to explain returns in a model that accounts for predicted returns based on information derived from variance. The second row indicates that variance retains power to explain returns after information from size is incorporated into the model. Therefore, Panel B of Table 2 shows that we are able to reject *ln(ME)* in favor of *Variance* as the true underlying factor in explaining the cross-section of stock returns. The results from the *J* test are consistent with the results from the portfolio return decomposition that attributes the source of outperformance in small-stock portfolios relative to large stock portfolios as originating from the variance of the underlying stocks.

Finally, we note that the average slope coefficient for own-stock variance in raw return regressions is close to one-half, as implied by Eq. (1). In also considering the result of no

statistically significant relationship between log returns and either *ln(ME)* or *Variance*, we conclude that the relationship between raw returns and stock variances simply reflects the mathematical description of stock returns in Eq. (1). That is, the use of arithmetic averages and raw returns rather than compound returns results in apparent relationships that have a *mathematical* rather than an *economic* explanation. When variables that are highly correlated with variance, such as size, are used in research methods that implicitly involve portfolios through the use of arithmetic averages, results must be interpreted with caution and the mathematical impact of variance must be considered.

5. Summary and Conclusion

We employ a mathematical parsing of portfolio compound returns into two key sources: 1) the average compound return of the portfolio's underlying stocks; and 2) the "excess growth rate" of the portfolio that is due to the underlying stocks' return variances and covariances. While the decomposition that we examine has been known to the literature for at least 25 years, it has been rarely cited; and, to our knowledge, we are the first to utilize this mathematical relationship in comparing individual stock characteristics to portfolio returns. We demonstrate that using the return components can yield new and meaningful insights into the sources of performance of commonly examined portfolios in the literature.

We provide new evidence regarding the oft-cited size, book-to-market, and momentum effects by examining the components of the returns to portfolios formed based on these factors.[10]

[10] In addition to sorting stocks based on underlying characteristics such as size, book-to-market ratios, and past returns, we also examine portfolios based on stocks' exposures to factors derived from such underlying characteristics. For example, to analyze the size effect, instead of sorting stocks directly based on market cap, we can first run a time-series regression of a stock's returns on the SMB portfolio returns and then sort by the coefficient estimate (i.e., factor loading) from this regression.

We show that small firms' average compound returns are lower than large firms' average compound returns. However, because small firms generally have higher return variances, portfolios of small stocks have higher excess growth rates than portfolios of large firms. It is the variance-induced excess growth rate that is primarily responsible for the outperformance of small stock portfolios relative to large stock portfolios. A similar effect is shown to exist for "contrarian" portfolios. In contrast, the performance of portfolios formed by sorting stocks by their book-to-market ratio is primarily determined by the performance of the underlying stocks. That is, the higher average compound returns of higher book-to-market stocks results in higher portfolio returns, overcoming the offsetting effect that excess growth rates are lower for these portfolios. Stocks displaying short-term momentum (based on 1-month to 2-year prior returns) also display patterns in portfolio returns that are generalizable to the underlying stocks in the portfolio.

Cross-sectional regressions of monthly raw returns and log returns confirm the results from the decomposition of portfolio returns. Specifically, the apparent cross-sectional relationship between size and stock raw returns is not robust to the inclusion of stock variance in the model, while the book-to-market effect is unchanged when variance is included. Furthermore, no cross-sectional relationship is evident between stock log returns and either size

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The factor-based portfolio returns that we observe are consistent with other studies of factor-based portfolios, such as Daniel and Titman (1997). Our portfolio return decomposition shows that the components of portfolio returns display similar patterns regardless of whether portfolios are formed directly from stock characteristics or indirectly based on factor exposures. We therefore do not tabulate results for portfolios formed on the basis of factor loadings.

Our method can also be used to examine additional factor-based portfolios, such as those derived from the liquidity measures of Pastor and Stambaugh (2003). For example, for portfolios formed from exposure to the level of the Pastor-Stambaugh liquidity measure, we find that portfolio returns are positively related to factor loadings across deciles, while the average underlying stock returns display a weak negative (and non-linear) association with the liquidity factor loadings. Therefore, portfolios formed based on exposure to this market-liquidity factor appear to be another example of a portfolio return pattern that is driven by the variance and excess growth rates of the underlying stocks, rather than the average compound returns of the underlying stocks.

or own-return variance, but a firm's book-to-market ratio is positively associated with its average log return.

The implications and applications of decomposing portfolio returns into effects due to stock returns and portfolio excess growth rates are wide-ranging. Whenever the returns of portfolios composed of high-volatility stocks are compared to benchmarks or control portfolios of lower-volatility stocks, we can expect to observe differences due to the excess growth rates of the portfolios. This potentially applies to any portfolio composed of relatively more volatile sample stocks, with the volatility arising due to measurement techniques (i.e., bid-ask bounce), the time period (i.e., January returns), firm characteristics (i.e. small size, high leverage, recent IPO activity, or low liquidity), or return patterns (i.e., momentum or contrarian). Even in the wider context of other social sciences that study phenomena subject to compound growth rates, any time the growth rate of a high-volatility treatment group is compared to a lower-volatility control group, there could be differences due to the excess growth effect. The current study demonstrates how the concept of an excess growth rate can explain portfolio returns in the context of several common characteristic-based portfolios. The ability of portfolio excess growth rates to explain other documented patterns in portfolio returns is a potentially valuable avenue for future research.

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Table 1
Size and Book-to-Market Decile Portfolios

This table reports the summary statistics for the estimates of the terms of Eq. (3) for equally weighted portfolios formed by size (market capitalization) book-to-market, beta, and past performance deciles for the sample period of January 1960 through December 2012. The terms for the weighted average return and the weighted average stock variance ($\sum_{i=1}^{N} w_i \bar{\gamma}_i$ and $\sum_{i=1}^{N} w_i \sigma_i^2$, respectively) are estimated each year using monthly data, with that year's estimate applied to each month within that year. The portfolio variance, σ_p^2, reflects the variance across all months in the sample period, while all other columns reflect the average of the given parameter across all months in the sample period. The portfolio's arithmetic average raw return, $\bar{r}_p$, is reported in the first column of estimates. The term $\bar{\gamma}_p(act.)$ reports the portfolio's average actual compound return (i.e., log return), while $\bar{\gamma}_p(est.)$ is the portfolio's estimated average compound return (i.e., log return) using Eq. (3). The remaining terms are defined in Eq. (3).

Portfolio	$\bar{r}_p$	$\bar{\gamma}_p(act.)$	$\bar{\gamma}_p(est.)$	$\sum_{i=1}^{N} w_i \bar{\gamma}_i$	$\sum_{i=1}^{N} w_i \sigma_i^2$	σ_p^2	$\bar{\gamma}_p^*(est.)$
Panel A: Size Deciles							
Low	0.0148	0.0121	0.0115	-0.0016	0.0308	0.0047	0.0130
2	0.0113	0.0090	0.0095	0.0020	0.0195	0.0045	0.0075
3	0.0121	0.0099	0.0118	0.0056	0.0166	0.0043	0.0062
4	0.0116	0.0096	0.0106	0.0051	0.0149	0.0040	0.0055
5	0.0117	0.0098	0.0111	0.0063	0.0132	0.0036	0.0048
6	0.0112	0.0095	0.0101	0.0060	0.0115	0.0032	0.0041
7	0.0114	0.0098	0.0110	0.0074	0.0102	0.0030	0.0036
8	0.0106	0.0091	0.0100	0.0066	0.0095	0.0028	0.0033
9	0.0104	0.0091	0.0096	0.0067	0.0083	0.0024	0.0029
High	0.0088	0.0077	0.0082	0.0058	0.0067	0.0020	0.0024
Panel B: Book-to-Market Deciles							
Low	0.0086	0.0055	0.0050	-0.0086	0.0335	0.0062	0.0136
2	0.0089	0.0065	0.0065	-0.0028	0.0235	0.0048	0.0094
3	0.0098	0.0076	0.0076	-0.0005	0.0204	0.0041	0.0081
4	0.0093	0.0074	0.0074	0.0000	0.0185	0.0037	0.0074
5	0.0115	0.0097	0.0099	0.0032	0.0170	0.0035	0.0067
6	0.0123	0.0105	0.0106	0.0040	0.0164	0.0033	0.0066
7	0.0135	0.0118	0.0118	0.0055	0.0157	0.0031	0.0063
8	0.0148	0.0131	0.0133	0.0066	0.0163	0.0030	0.0066
9	0.0165	0.0146	0.0145	0.0071	0.0183	0.0035	0.0074
High	0.0213	0.0184	0.0185	0.0071	0.0276	0.0049	0.0114

Portfolio	$\bar{r}_p$	$\bar{\gamma}_p(act.)$	$\bar{\gamma}_p(est.)$	$\sum_{i=1}^{N} w_i \bar{\gamma}_i$	$\sum_{i=1}^{N} w_i \sigma_i^2$	σ_p^2	$\bar{\gamma}_p^*(est.)$
Panel C: Beta Deciles							
Low	0.0104	0.0103	0.0102	0.0030	0.0152	0.0008	0.0072
2	0.0102	0.0101	0.0102	0.0048	0.0117	0.0010	0.0053
3	0.0112	0.0110	0.0110	0.0055	0.0124	0.0015	0.0055
4	0.0107	0.0106	0.0107	0.0047	0.0137	0.0019	0.0059
5	0.0106	0.0104	0.0105	0.0041	0.0152	0.0023	0.0065
6	0.0112	0.0109	0.0110	0.0039	0.0170	0.0028	0.0071
7	0.0107	0.0104	0.0104	0.0027	0.0191	0.0035	0.0078
8	0.0105	0.0101	0.0103	0.0012	0.0227	0.0044	0.0091
9	0.0103	0.0099	0.0099	-0.0007	0.0272	0.0059	0.0107
High	0.0113	0.0104	0.0102	-0.0054	0.0424	0.0112	0.0156
Panel D: Prior Short-term (6-month) Performance Deciles							
Low	0.0092	0.0086	0.0092	-0.0131	0.0522	0.0076	0.0223
2	0.0082	0.0078	0.0078	-0.0045	0.0292	0.0047	0.0123
3	0.0089	0.0086	0.0086	-0.0003	0.0214	0.0035	0.0089
4	0.0094	0.0092	0.0091	0.0020	0.0171	0.0028	0.0071
5	0.0101	0.0099	0.0098	0.0037	0.0145	0.0024	0.0061
6	0.0106	0.0104	0.0104	0.0049	0.0131	0.0021	0.0055
7	0.0113	0.0111	0.0110	0.0054	0.0133	0.0021	0.0056
8	0.0109	0.0107	0.0107	0.0048	0.0142	0.0024	0.0059
9	0.0107	0.0105	0.0104	0.0034	0.0172	0.0031	0.0070
High	0.0094	0.0090	0.0090	-0.0021	0.0271	0.0049	0.0111
Panel E: Prior Long-term (5-year) Performance Deciles							
Low	0.0162	0.0155	0.0161	-0.0078	0.0559	0.0081	0.0239
2	0.0103	0.0099	0.0098	-0.0044	0.0333	0.0048	0.0142
3	0.0095	0.0092	0.0092	-0.0011	0.0242	0.0037	0.0103
4	0.0091	0.0089	0.0089	0.0010	0.0187	0.0029	0.0079
5	0.0094	0.0092	0.0091	0.0027	0.0152	0.0023	0.0064
6	0.0101	0.0099	0.0099	0.0041	0.0135	0.0021	0.0057
7	0.0094	0.0092	0.0092	0.0039	0.0127	0.0022	0.0053
8	0.0098	0.0096	0.0096	0.0043	0.0128	0.0023	0.0053
9	0.0090	0.0088	0.0088	0.0029	0.0144	0.0027	0.0059
High	0.0065	0.0063	0.0063	-0.0015	0.0197	0.0042	0.0078

Interpretation: Table 1 shows how portfolio compound returns can differ from the underlying components of those returns for portfolios formed from several stock characteristics. Panel A shows that portfolio returns ($\bar{\gamma}_p(act.)$) are negatively associated with market-cap even though

average underlying stock returns ($\sum_{i=1}^{N} w_i \bar{\gamma}_i$) are positively related to market cap. Panel B shows that both portfolio returns and average stock returns are positively associated with book-to-market ratios. Panel C displays results consistent with both portfolios and average stock returns having only weak and non-linear associations with estimates of beta. Panel D shows that short-term past returns have a positive but non-linear association with both portfolio returns and average stock returns, while Panel E shows that long-term past returns are positively associated with average stock returns but negatively associated with portfolio returns.

Table 2
Average Slope Coefficients from Cross-Sectional Regressions

In Panel A. each row reports the time-series average slope coefficients from cross-sectional regressions for the sample period of January 1960 through December 2012. Monthly stock raw returns and compound returns (i.e., log returns) are used as dependent variables, with the beginning-of-year market cap (*ln(ME)*), beginning-of-year book-to-market ratio (*ln(BE/ME)*), prior 5-year beta estimate, and same-year log return variance (*Variance*) used as independent variables. Rows one through four each report results from univariate regression models, while row five reports results from a multivariate regression model that includes all four independent variables. Panel B reports the second stage results from a *J Test* (Davidson and MacKinnon, 1981) of *ln(ME)* and *Variance* in the cross-sectional regressions. In the first stage, returns are regressed on *ln(ME)* or *Variance.* In the second stage, the predicted values of returns ($\hat{r}$) are used along with the actual values of *ln(ME)* (row 1) or *Variance* (row 2) to explain actual returns. T-statistics are reported in parentheses below the coefficient estimates.

Panel A: Average Slope Coefficients

Dependent Variable: Raw Returns				Dependent Variable: Log Returns			
ln(ME)	*ln(BE/ME)*	*Variance*	*Beta*	*ln(ME)*	*ln(BE/ME)*	*Variance*	*Beta*
-0.0014				0.0009			
(-3.20)				(2.28)			
	0.0040				0.0049		
	(4.98)				(6.42)		
		0.3597				-0.1633	
		(3.72)				(-1.81)	
			0.0022				-0.0038
			(1.33)				(-2.32)
-0.0002	0.0026	0.4413	-0.0012	0.0001	0.0025	-0.0708	-0.0014
(-0.06)	(3.92)	(5.03)	(-0.90)	(0.26)	(3.90)	(-0.87)	(-1.06)

Panel B: J Test of ln(ME) and Variance

Dependent Variable: Raw Returns			
ln(ME)	*$\hat{r}$(Variance)*	*Variance*	*$\hat{r}$(ln(ME))*
0.0031	1.8716		
(0.12)	(22.79)		
		0.2950	0.8108
		(3.77)	(2.07)

Interpretation: Panel A of Table 2 compares the usefulness of stocks' market caps (*ln(ME)*), book-to-market ratios (*ln(BE/ME)*), variances, and betas, to explain the cross-section of both raw returns and compound (log) returns. The multivariate regression coefficients reported in Row 4 show that variance, but not market cap, is significant in explaining raw returns, although variance is insignificant in explaining log returns. This is consistent with a mechanical relationship between variance and raw returns. The J-test of Panel B confirms that stock variance explains the cross-section of stock returns better than market cap.